Chirality inducing G_4-flux in F-theory compactifications

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Introduction

In this talk I will present results from our recent Paper

[Krause, Mayrhofer, Weigand; 1109.3454 [hep-th]]

Main results:

- We construct a globally defined non-Cartan G_4-flux in F-Theory.
- We show that it induces chirality and compute the chiral index.
- We exemplify this in a 3-generation F-theory GUT compactification.

Indpendent, but related recent work:

- [Braun, Collinucci, Valandro; 1107.5337 [hep-th]]: similar fluxes in SU(2) F-theory models, but with different resolution techniques
- [Marsano, Schäfer-Nameki 1108.1794 [hep-th]]: global extension of spectral cover fluxes in $SU(5)$ models without $U(1)$ restriction (less global sensitivity)

Introduction

Structure of this talk:

- overview of F-Theory
- introduction of a non-Cartan G_4-flux in $SU(5) \times U(1)_X$ models
- presentation of chirality relation

F-Theory (1)

F-Theory can be viewed as a non-perturbative extension of Type IIB-theory, in which the axio-dilaton is geometrized as a torus.

In particular, F-Theory models live on elliptically fibred Calabi-Yau four-folds:

$$\begin{array}{ccc} T & \hookrightarrow & Y_4 \\ & & \downarrow \\ & & B_3 \end{array}$$

F-Theory can also be viewed as dual to M-Theory via reduction of one of the torus circles and performance of T-duality along the other.

In particular, the bulk and brane fluxes of F-Theory are encoded in the G_4-flux of M-Theory.

F-Theory (2)

To construct a brane set-up in F-Theory, in which strings are in the fundamental representation of $SU(5)$, **24**, the complex structure moduli are restricted so as to induce an A_4-singularity [Bershadsky et. al.; 9605200 [hep-th]]:

(almost) generic fibre (in $\mathbb{P}_{231}[x, y, z]$):

$$P_T = \{y^2 + a_1\, x\, y\, z + a_3\, y\, z^3 = x^3 + a_2\, x^2\, z^2 + a_4\, x\, z^4 + a_6\, z^6\}$$

where the a_i depend on the base.

Restriction:

$$a_1 = a_1, \quad a_2 = a_{2,1}\, w \quad a_3 = a_{3,2}\, w^2 \quad a_4 = a_{4,3}\, w^3 \quad a_6 = a_{6,5}\, w^5$$

$\rightarrow SU(5)$-singularity in the fibre over the GUT surface $w = 0$

F-Theory (3)

For the standard model extension $SU(5) \to SU(3) \times SU(2) \times U(1)$ one further would like states in the **10**- and the **5**-representation.

$\rightarrow$ These occur on Enhancement Curves.

Correspondingly, the Yukawa coupling are encoded in Enhancement Points, where these curves meet.

Additional $U(1)$

For generic $SU(5)$-models only a single **5**-curve occurs.

However, one would like these to split into $\mathbf{5}_m$ and $\mathbf{5}_H$.

To enforce the splitting, the complex structure moduli are further restricted to $a_6 = 0$.

As was shown in [Grimm, Weigand; 1006.0226 [hep-th]], this induces an additional $SU(2)$-singularity along the curve $a_3 = a_4 = 0$.

Type IIB picture:
States on this curve live in the $U(1) \times U(1)$; the diagonal $U(1)$ is projected out by involution, leaving one additional $U(1)$.

Resolution via Blow-up

Reliable calculations of topological or geometric properties require the singularities to be resolved.

$SU(5)$-singularity: 4 new divisors e_i; 4 new divisor classes E_i.
$SU(2)$-singularity: 1 new divisor s; 1 new divisor class S.
$\leftrightarrow$ extra $U(1)$ symmetry from expansion $C_3 = A \wedge (S + \dots)$

	x	y	z	s	e_1	e_2	e_3	e_4	e_0
W	·	·	·	·	·	·	·	·	1
c_1	2	3	·	·	·	·	·	·	·
Z	2	3	1	·	·	·	·	·	·
S	-1	-1	·	1	·	·	·	·	·
E_1	-1	-1	·	·	1	·	·	·	-1
E_2	-2	-2	·	·	·	1	·	·	-1
E_3	-2	-3	·	·	·	·	1	·	-1
E_4	-1	-2	·	·	·	·	·	1	-1

Flux (1)

Chiral matter spectrum requires G_4-flux.

Obvious candidate for flux in presence of $U(1)$:

$$C_3 = A \wedge (S + \ldots) \quad \rightarrow \quad G_4 = F \wedge (S + \ldots)$$

General conditions from the dual M-theory picture ('one leg in the fibre, three legs in the base') [Denef; 0803.1194 [hep-th]]

$$\int_{\tilde{Y}_4} G_4 \wedge D_a \wedge D_b = 0$$
$$\int_{\tilde{Y}_4} G_4 \wedge Z \wedge D_a = 0$$

Conditions met by e.g.

$$G_4 = [E_i] \wedge F_i \qquad \text{(Cartan Fluxes)}$$
$$G_4 = [(Z + \bar{K} - S)] \wedge F_X$$

Flux (2)

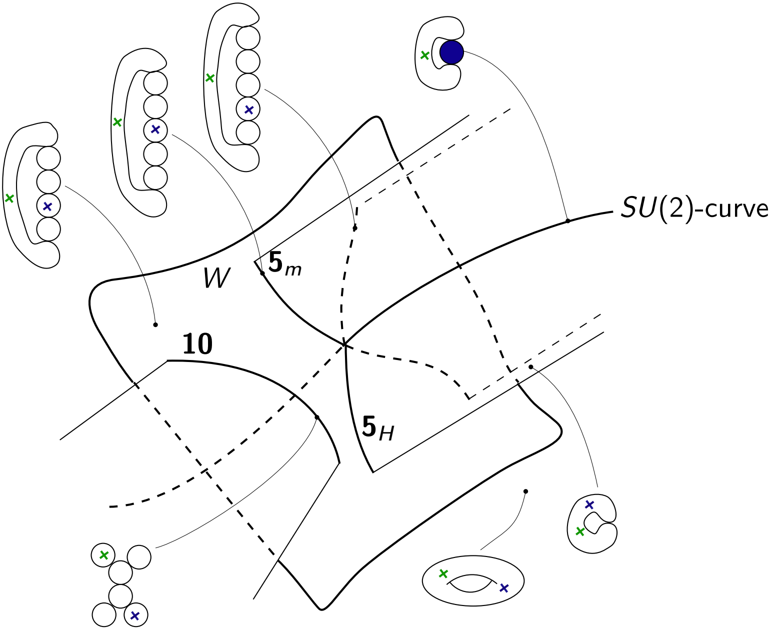
To construct a non-Cartan flux, one further requires

$$\int_{\tilde{Y}_4} G_4 \wedge E_i \wedge D_b = 0$$

Conditions met by ($G_4 = \mathrm{w}_X \wedge F_X$):

$$\mathrm{w}_X = 5 \left(Z + \bar{K} - S \right) - a_i E_i \quad (1)$$

with $a_i = (2, 4, 6, 3)$.



$SU(2)$-curve
W
$\mathbf{5}_m$
$\mathbf{10}$
$\mathbf{5}_H$

Chirality (1)

In Type IIB, the chirality of states on a curve of intersecting branes is given by

$$q \int_{\mathcal{C}_{R_q}} F_X$$

where $\mathcal{C}_{R_q}$ denotes the curve and R_q the group representation.

One would like to relate this to the integral of a four-form flux over the matter surfaces in $\tilde{Y}_4$ associated to $\mathcal{C}_{R_q}$.

These surfaces, C_{R_q}, consist of a linear combination of the blow-up $\mathbb{P}^1$s fibred over the enhancement curve $\mathcal{C}_{R_q}$.

The linear combination is such that in the dual M-theory picture, an M2-brane wrapping this combination is in one of the states of the representation R_q.

Chirality (2)

With the non-Cartan flux constructed above one finds

$$\int_{C_{R_q}} G_4 = \int_{C_{R_q}} w_X \wedge F_X = q \int_{\mathcal{C}_{R_q}} F_X \tag{2}$$

with q the $U(1)_X$-charge.

Summary and Outlook

We have

- found $U(1)$-induced, non-Cartan flux in $SU(5) \times U(1)_X$ models
- demonstrated that this induces chirality

Further results not presented here, but included in the paper:

- Computed induced D3-brane tadpole and D-term supersymmetry condition
- Implemented in global F-theory $SU(5) \times U(1)_X$ compactification with 3 chiral generations
- For recombination of the **5**-curves/ general $SU(5)$-models, can define G_4 via horizontal four-forms, see also [Braun, Collinucci, Valandro; 1107.5337 [hep-th]]. The chirality formulae change only slightly.

In the future we hope to better understand

- the quantization conditions imposed on the flux derived above,
- the direct link between the above flux and Type IIB-fluxes.

Questions

Thank you for your attention!

Towards the Matter Surfaces: $\mathbb{P}^1$-structure

Generarically, a $\mathbb{P}^1$ is given in the ambient five-fold by

$$P_T|_{e_i=0} \cap e_i \cap y_a \cap y_b$$

$$i \in \{0,1,2,3,4\} \quad \Rightarrow \quad 5\ \mathbb{P}^1\text{s of } SU(5)$$

Over enhancement curve (e.g. $y_a = a_1$), $P_T|_{e_i=0}$ may factorise. Take, e.g. $P_T|_{e_i=0} = A\,B$. $\Rightarrow$ 2 $\mathbb{P}^1$s from e_i:

$$A \cap e_i \cap a_1 \cap y_b$$
$$B \cap e_i \cap a_1 \cap y_b$$

In this way one finds the additional $\mathbb{P}^1$s for necessary for $SO(10)$-, $SU(6)$-enhancements, etc..

Note: One does not obtain the $\tilde{E}_6$-structure in this way, see [Esole, Yau].

Matter Surfaces

Multiplicities and Intersection structure of the $\mathbb{P}^1$s allow one to

→ calculate Cartan charge of each $\mathbb{P}^1$
e.g. $(1, 0, 0, -1)$

→ determine the group theoretic representation of each $\mathbb{P}^1$
(more concretely: of an M2-brane wrapping a certain $\mathbb{P}^1$)
e.g. $\mu_{10} - \alpha_2 - \alpha_3 - \alpha_4$

→ express each group theoretic state as a linear combination of $\mathbb{P}^1$s
e.g. $\mu_{10} \simeq \mathbb{P}^1_{0A} + \mathbb{P}^1_{14} + \mathbb{P}^1_{4D}$

→ define the matter surfaces accordingly
e.g. $C^1_{10} =$ 'linear combination of $\mathbb{P}^1$s corresponding to μ_{10}
fibred over $\mathcal{C}_{10} := \{a_1 = 0\}$'