Primordial decays and non-Gaussianities.

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Introduction

- Friendship with someone different from us is possible, e.g. particle physics and cosmology!

- Particle physics provides motivated models for high energy physics. On the other hand, cosmological data provide tests and constrains, e.g from BBN, the spectrum of CMB fluctuations, Non-Gaussianities...

Homogeneous universe

Perturbations II order

Perturbations I order

Decays: how they affect primordial perturbations

Non-linear perturbations

Let us consider a cosmological perfect fluid:

$$T_{ab} = (\rho + P)\, u_a u_b + P g_{ab} .$$

From the energy-momentum tensor conservation, it follows that the perturbations

$$\zeta_{_A} = \delta\mathcal{N} + \frac{1}{3(1 + w_{_A})} \ln\left(\frac{\rho_{_A}}{\bar{\rho}_{_A}}\right) \ ,$$

are conserved for adiabatic fluids, such that: $P = w\rho$. In the presence of several fluids we can define:

$$\zeta_A = \zeta_B = \zeta_r \qquad \text{(adiabatic mode)}$$

$$S_A \equiv 3(\zeta_A - \zeta_r) \qquad \text{(isocurvature modes)}$$

Evolution of perturbations

Let us focus on the decay of some species σ. What is its impact on the primordial perturbations?

- Sudden decay approximation: $H_d = \Gamma_\sigma$.
- On the decay hypersurface (uniform energy) $\delta\mathcal{N} = \zeta$.
- Decay in several species with branching ratios:

$$\gamma_{A\sigma} \equiv \frac{\Gamma_{A\sigma}}{\Gamma_\sigma}, \qquad \Gamma_\sigma \equiv \sum_A \Gamma_{A\sigma}\,.$$

- Arbitrary EOS $w = P/\rho$ for all the involved fluids.

The final goal is the calculation of the ζ_{A+} after the decay, as a function of the ζ_{A-} before the decay.

After the decay

From the energy conservation:

$$\sum_A \bar{\rho}_{A-} e^{3(1+w_A)(\zeta_{A-}-\zeta)} = \bar{\rho}_{\rm decay} = \sum_B \bar{\rho}_{B+} e^{3(1+w_B)(\zeta_{B+}-\zeta)},$$

It follows that the perturbations at third order are given by

$$\zeta_{A+} = \sum_B T_A^{\ B} \zeta_{B-} + \sum_{B,C} U_A^{BC} \zeta_{B-}\zeta_{C-} + \sum_{B,C,D} V_A^{BCD} \zeta_{B-}\zeta_{C-}\zeta_{D-},$$

where the coefficients of T, U and V are functions of:

$$w_\sigma \qquad w_B \qquad \gamma_{B\sigma} \qquad \bar{\rho}_\sigma \qquad \bar{\rho}_B \qquad (B \neq \sigma)$$

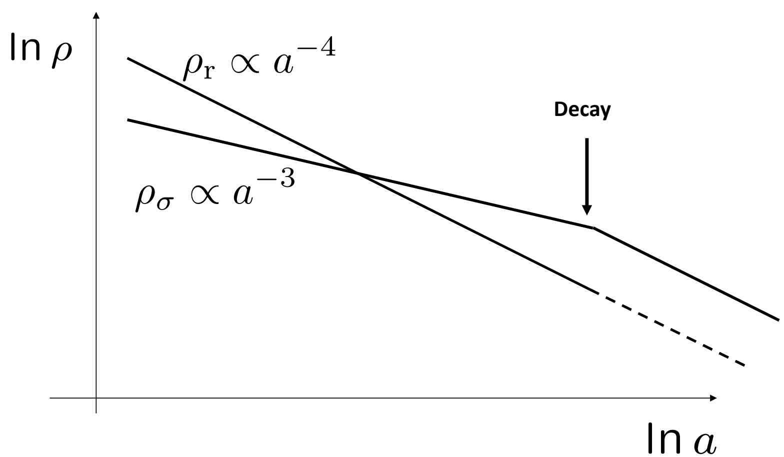
That is, they only depend on the homogeneous parameters!

Langlois & AL (2010), Langlois &Takahashi (2010)

An application:
the curvaton model

The curvaton

A curvaton σ is a light scalar field during inflation ($m < H$). When $m \sim H$ it oscillates and eventually decays.



Linde & Mukhanov (1996),
Enqvist & Sloth (2001),
Lyth & Wands(2001)

- Mixed inflaton-curvaton perturbations. Langlois & Vernizzi (2004)
- Production of isocurvature perturbations: Lyth & Wands (2003)

$$S_A \equiv 3(\zeta_A - \zeta_r)$$

Calculation of perturbations

Let us consider radiation (r), cold dark matter (c) and a curvaton (σ).

- Perturbation from the inflaton decay: ζ_{inf};
- Curvaton entropy perturbation: $S_\sigma = \hat{S} - \frac{1}{4}\hat{S}^2 + \frac{1}{12}\hat{S}^3$
- Before the decay $\zeta_{c-} = \zeta_{r-} = \zeta_{inf}$.

The curvaton decay yields:

$$\zeta_r = \zeta_{\rm inf} + z_1 \hat{S} + \frac{1}{2} z_2 \hat{S}^2 + \frac{1}{6} z_3 \hat{S}^3 \,,$$

$$S_c = s_1 \hat{S} + \frac{1}{2} s_2 \hat{S}^2 + \frac{1}{6} s_3 \hat{S}^3 \,,$$

where the coefficients z and s depend on $\gamma_{A\sigma}$, Ω_σ, Ω_A through:

$$f_A \equiv \frac{\gamma_{A\sigma}\Omega_\sigma}{\Omega_A + \gamma_{A\sigma}\Omega_\sigma} \qquad r \equiv \xi\,\tilde{r} \qquad \xi \equiv \frac{f_r}{\Omega_\sigma} \qquad \tilde{r} \equiv \frac{3\Omega_\sigma}{4 - \Omega_\sigma}$$

Langlois & AL (2010), Langlois &Takahashi (2010)

Power spectrum

For a generic perturbation ζ, the power spectrum is defined as:

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle = (2\pi)^3 \delta^{(3)}(k_1 + k_2) P_\zeta(k_1) \qquad P_\zeta(k) = \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k)$$

The power spectrum of $\hat{S}$, generated during inflation, is given by

$$\mathcal{P}_{\hat{S}}(k) = \frac{4}{\sigma_*^2} \left(\frac{H_*}{2\pi} \right)^2$$

In our model:

$$\mathcal{P}_{\zeta_{\rm r}} = \mathcal{P}_{\zeta_{\rm inf}} + \frac{r^2}{9} \mathcal{P}_{\hat{S}} \equiv \Xi^{-1} \frac{r^2}{9} \mathcal{P}_{\hat{S}}$$

$$\mathcal{P}_{S_c} = (f_c - r)^2 \mathcal{P}_{\hat{S}} \, ,$$

where Ξ is the fraction of the power spectrum due to the curvaton and f_c, r depend on $\gamma_{A\sigma}, \Omega_\sigma, \Omega_A$.

Non-adiabaticity

Perturbations are mostly adiabatic: $\alpha \equiv \frac{\mathcal{P}_{S_c}}{\mathcal{P}_{\zeta_{\rm r}}} \ll 1\,,$

depending on the correlation: $\mathcal{C} \equiv \frac{\mathcal{P}_{S_c,\zeta_r}}{\sqrt{\mathcal{P}_{S_c}\mathcal{P}_{\zeta_r}}}\,.$

Constraints at 95% C.L.

- $\mathcal{C}=0$, e.g. axion: $\alpha < 0.064$.
- $\mathcal{C}=1$, e.g. ”pure” curvaton: $\alpha < 0.0037$.

Komatsu et al. (2010)

In our model:

$$\alpha = 9\left(1-\frac{f_c}{r}\right)^2 \Xi \qquad\qquad \mathcal{C} = \mathrm{sgn}(f_c - r)\sqrt{\Xi}$$

Hence we need $\Xi \ll 1$ or $|f_c - r| \ll r$.

NG - adiabatic case

NG of local type arise from a perturbation of the kind:

$$\zeta = \phi + \frac{3}{5} f_{NL}^{(\mathrm{local})} \phi^2 \,,$$

The bispectrum of ζ is defined as:

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 \delta(\Sigma_i \vec{k}_i) \frac{6}{5} f_{NL}^{(\mathrm{local})} \left[P_{\zeta_r}(k_1) P_{\zeta_r}(k_2) + \mathrm{perms} \right]$$

Constraints at 95% C.L.

$$-10 \leq f_{NL}^{(\mathrm{local})} \leq 74$$

Komatsu et al. (2010)

Detection of significant f_{NL} would rule out the simplest models of inflation.

NG - beyond adiabaticity

When several observable quantities X^I are present:

$$X^I = N^I_a \phi^a + \frac{1}{2} N^I_{ab} \phi^a \phi^b + \ldots ,$$

where the ϕ^a are Gaussian random fields, such that

$$\langle \phi^a(\vec{k}) \phi^b(\vec{k}') \rangle = (2\pi)^3 \, P^{ab}(k) \, \delta(\vec{k} + \vec{k}') \, ,$$

we can define the generalized bispectra:

$$\langle X^I_{\vec{k}_1} X^J_{\vec{k}_2} X^K_{\vec{k}_3} \rangle = (2\pi)^3 \delta(\Sigma_i \vec{k}_i) B^{IJK}(k_1, k_2, k_3) \, .$$

In our case we have $X^I = \zeta, S_c$.

The bispectrum

In our case $X^I = \zeta, S_c$ and we have only one DOF $\hat{S}$:

$$\zeta_r = \zeta_{\rm inf} + z_1 \hat{S} + \frac{1}{2} z_2 \hat{S}^2 + \dots \qquad S_c = s_1 \hat{S} + \frac{1}{2} s_2 \hat{S}^2 + \dots$$

It follows that the generalized bispectrum takes the form:

$$B^{IJK}(k_1, k_2, k_3) = b_{NL}^{I,JK} P_{\hat{S}}(k_2) P_{\hat{S}}(k_3) + b_{NL}^{J,KI} P_{\hat{S}}(k_1) P_{\hat{S}}(k_3) + b_{NL}^{K,IJ} P_{\hat{S}}(k_1) P_{\hat{S}}(k_2),$$

with

$$b_{NL}^{I,JK} \equiv N_{(2)}^I N_{(1)}^J N_{(1)}^K,$$

$$N_{(2)}^{\zeta} = z_2, \quad N_{(2)}^S = s_2, \quad N_{(1)}^{\zeta} = z_1, \quad N_{(1)}^S = s_1$$

Hence NG is quantified through six independent parameters!

NG for the curvaton model

The b parameters are proportional to the "standard" f parameters

$$\tilde{f}_{NL}^{I,JK} \equiv \frac{6}{5} f_{NL}^{I,JK} = \left(\frac{P_{\hat{S}}}{P_\zeta}\right)^2 b_{NL}^{I,JK},$$

where

$$\left(\frac{P_{\hat{S}}}{P_\zeta}\right)^2 = \frac{\Xi^2}{z_1^4}$$

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It follows:

$$\tilde{f}_{NL}^{\zeta,\zeta\zeta} = \frac{z_2}{z_1^2}\,\Xi^2, \quad \tilde{f}_{NL}^{\zeta,\zeta S} = \frac{s_1 z_2}{z_1^3}\,\Xi^2, \quad \tilde{f}_{NL}^{\zeta,SS} = \frac{s_1^2 z_2}{z_1^4}\,\Xi^2,$$

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For instance the purely adiabatic coefficient is:

$$\tilde{f}_{NL}^{\zeta\zeta\zeta} = \left(\frac{3}{2r} + \frac{2}{\xi} - 4 - \frac{r}{\xi^2}\right)\Xi^2$$

The parameter space

The relevant parameters are:

- The ratio between the curvaton and the inflaton contributions to the radiation spectrum $\mathcal{P}_{\zeta_r}$:

$$\lambda \equiv \frac{(r^2/9)\mathcal{P}_{\hat{S}}}{\mathcal{P}_{\zeta_{\rm inf}}} = \frac{\Xi}{1-\Xi}$$

- The fraction of CDM generated by the curvaton decay:

$$f_c \equiv \frac{\gamma_{c\sigma}\Omega_\sigma}{\Omega_c + \gamma_{c\sigma}\Omega_\sigma}$$

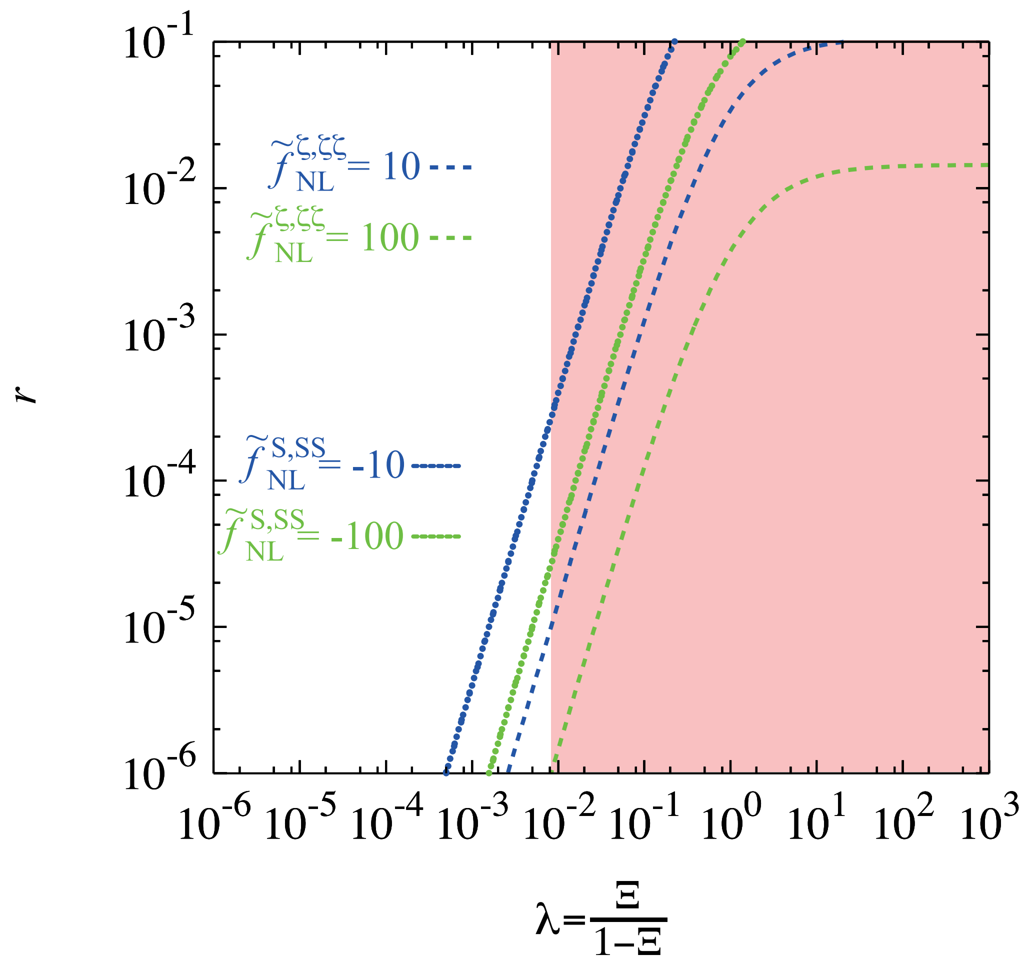
- The transfer efficiency times energy fraction:

$$r = \left(\frac{\gamma_{r\sigma}}{1-(1-\gamma_{r\sigma})\Omega_\sigma}\right)\left(\frac{3\Omega_\sigma}{4-\Omega_\sigma}\right)$$

The parameter space

CASE 1: $f_c = 0$

- Pink region: ruled out.
- $\lambda \ll 1$ is required.
- Relevant NG only for small r.
- $f_{NL}^{\zeta\zeta,\zeta} \propto \alpha^2 r^{-1}$.
- $\tilde{f}_{NL}^{I,JK} \simeq (-3)^{I_S} \tilde{f}_{NL}^{\zeta,\zeta\zeta}$, I_S is the number of S among the indices.

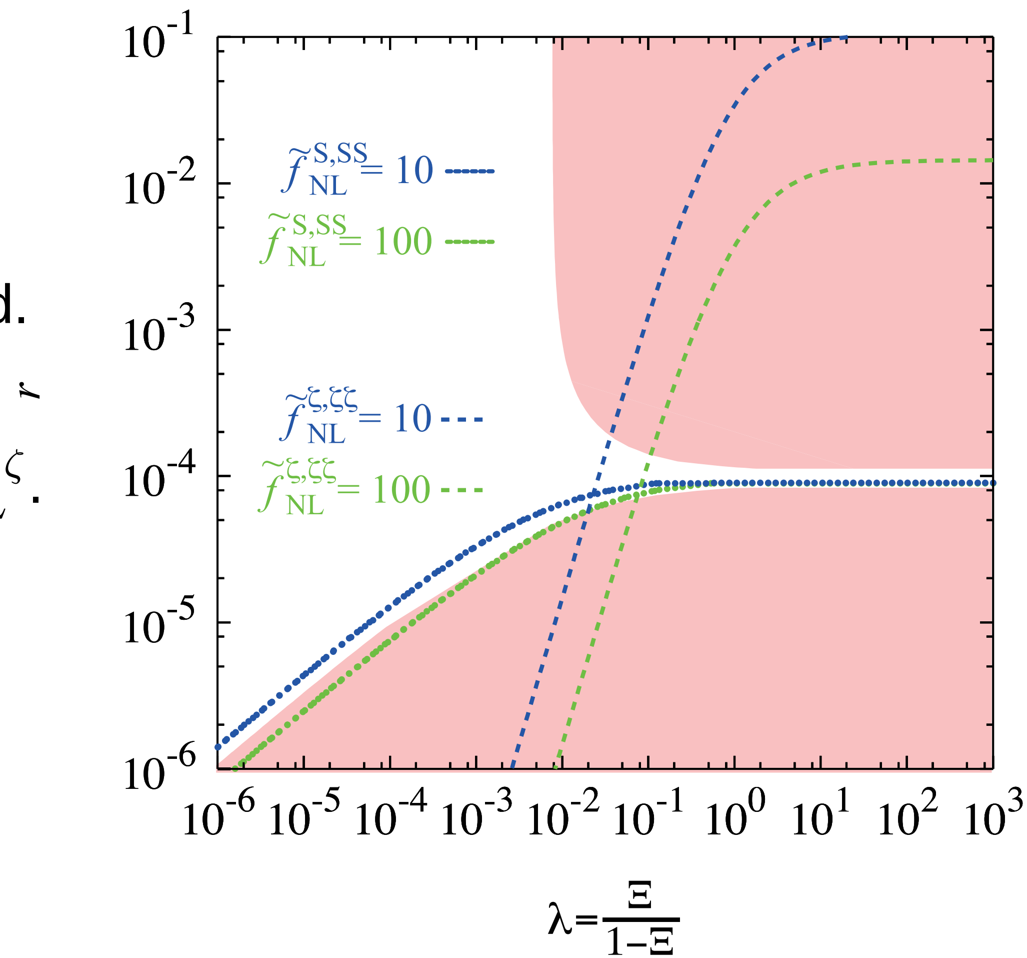


Picture from Langlois & Takahashi (2010)

The parameter space

CASE 1: $f_c = 10^{-4}$

- Pink region: ruled out.
- New region at $\lambda \sim 1$
  - $|f_c - r| \ll r$ required.
  - $f_{NL}^{\zeta\zeta,\zeta}$ dominates.
  - $f_{NL}^{S,\zeta,\zeta}$ may be $\simeq f_{NL}^{\zeta\zeta,\zeta}$.
- Region $\lambda \ll 1$
  - When $r \ll f_c \ll 1$, $f_{NL}^{S,SS}$ dominates.



Picture from Langlois & Takahashi (2010)

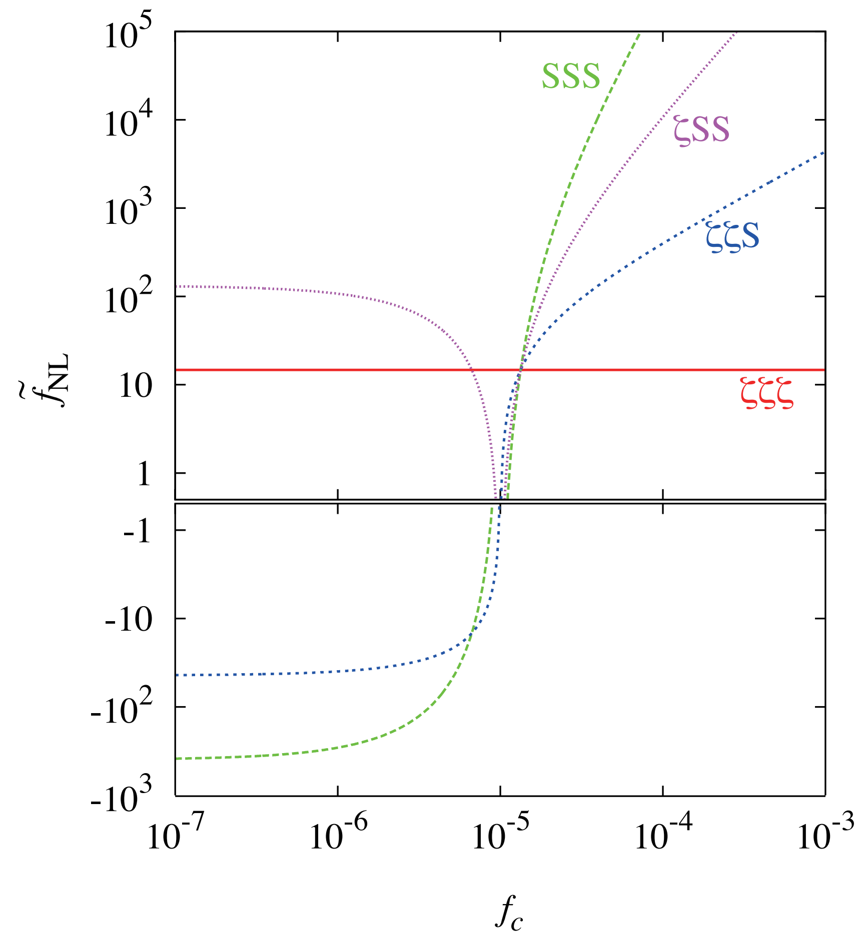
The parameter space

Case 3: $r = 10^{-5}$, $\lambda = 10^{-3}$: three regimes are shown:

- $f_c \ll r \ll 1$
  $\tilde{f}_{NL}^{I,JK} \simeq (-3)^{I_S} \tilde{f}_{NL}^{\zeta,\zeta\zeta}$,
  I_S = number of S
  among the indices.

- $|f_c - r| \ll r$
  $f_{NL}^{\zeta\zeta,\zeta}$ dominates.

- $r \ll f_c \ll 1$
  $f_{NL}^{S,SS}$ dominates.



Picture from Langlois & Takahashi (2010)

Conclusions

- It is possible to treat systematically linear and non linear cosmological perturbations.
- This generic approach can be used in a wide range of models.
- As an example, a model where one or two curvatons participate together with the inflaton to the production of perturbations is analyzed.
- In the presence of isocurvature modes, local NG are parametrized through six independent parameters.
- The set of six independent NG parameters can be constrained using CMB data.

The end

Thank you for your kind attention

The parameter space

The relevant parameters are:

- The ratio between the curvaton and the inflaton contributions to the radiation spectrum $\mathcal{P}_{\zeta_r}$:

$$\lambda \equiv \frac{(r^2/9)\mathcal{P}_{\hat{S}}}{\mathcal{P}_{\zeta_{\rm inf}}} = \frac{\Xi}{1-\Xi}$$

- The fraction of CDM generated by the curvaton decay:

$$f_c \equiv \frac{\gamma_{c\sigma}\Omega_\sigma}{\Omega_c + \gamma_{c\sigma}\Omega_\sigma}$$

- The transfer efficiency times energy fraction:

$$r = \left(\frac{\gamma_{r\sigma}}{1-(1-\gamma_{r\sigma})\Omega_\sigma}\right)\left(\frac{3\Omega_\sigma}{4-\Omega_\sigma}\right)$$

Scenario with two curvatons

As a second application of our formalism we considered a model where two curvatons, σ and χ, decay in radiation and cold dark matter:

$$\zeta_{\rm r} = \zeta_{\rm r0} + z_\sigma \hat{S}_\sigma + z_\chi \hat{S}_\chi + z_{\sigma\chi}\hat{S}_\sigma \hat{S}_\chi + \frac{1}{2} z_{\sigma\sigma}\hat{S}_\sigma^2 + \frac{1}{2} z_{\chi\chi}\hat{S}_\chi^2$$

$$S_c = s_\sigma \hat{S}_\sigma + s_\chi \hat{S}_\chi + s_{\sigma\chi}\hat{S}_\sigma \hat{S}_\chi + \frac{1}{2} s_{\sigma\sigma}\hat{S}_\sigma^2 + \frac{1}{2} s_{\chi\chi}\hat{S}_\chi^2$$

Since the spectra of σ and χ are independent, $P_{S_\chi} \equiv \Lambda P_{S_\sigma}$ we end up with six independent NG parameters:

$$b_{NL}^{I,JK} \equiv N^I_{\sigma\sigma} N^J_\sigma N^K_\sigma + \Lambda N^I_{\sigma\chi}\left(N^J_\sigma N^K_\chi + N^J_\chi N^K_\sigma\right) + \Lambda^2 N^I_{\chi\chi} N^J_\chi N^K_\chi \,,$$

Towards observations

We can define the reduced bispectrum $b_{l_1 l_2 l_3}$:

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = \mathcal{G}^{m_1 m_2 m_3}_{l_1 l_2 l_3} b_{l_1 l_2 l_3} ,$$

where a_{lm} are the coefficients of the expansion:

$$\frac{\Delta T(\hat{n})}{T} = \sum_{lm} a_{lm} Y_{lm}(\hat{n}) \, .$$

The transfer function $g^I_l(k)$, enables us to write:

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3\vec{k}}{(2\pi)^3} \left(\sum_I X^I(\vec{k}) g^I_l(k) \right) Y^*_{lm}(\hat{\vec{k}}).$$

Towards observations - bispectrum

Hence we can express the bispectrum as:

$$b_{l_1 l_2 l_3} = 3 \sum_{I,J,K} N^I_{ab} N^J_c N^K_d \int_0^\infty r^2 dr \tilde{\beta}^I_{(l_1}(r) \beta^{J,ac}_{l_2}(r) \beta^{K,bd}_{l_3)}(r),$$

with

$$\tilde{\beta}^I_l(r) \equiv \frac{2}{\pi} \int k^2 dk j_l(kr) g^I_l(k), \qquad \beta^{I,ab}_l(r) \equiv \frac{2}{\pi} \int k^2 dk j_l(kr) g^I_l(k) P^{ab}(k)\,.$$

It follows that isocurvature and mixed NG modes can be constrained by using CMB data (Langlois & van Tent, 2011).

The curvaton

Before its decay, the curvaton σ obeys:

$$\rho_\sigma \quad = \quad m^2\sigma^2\,.$$

Its inhomogeneous energy density on a spatially flat hypersurface is:

$$\bar{\rho}_\sigma e^{S_\sigma} \quad = \quad m^2\left(\bar{\sigma}+\delta\sigma\right)^2\;,$$

from which follows that the curvaton entropy perturbation contains a linear gaussian part $\hat{S}$ and a non-linear part:

$$S_\sigma = \hat{S} - \frac{1}{4}\hat{S}^2 + \frac{1}{12}\hat{S}^3\,, \qquad \text{with} \qquad \hat{S} \equiv 2\frac{\delta\sigma_*}{\bar{\sigma}_*}$$

the $*$ indicates the epoch of Hubble radius crossing.

The z and s coefficients

$$\zeta_r = \zeta_{\rm inf} + z_1 \hat{S} + \frac{1}{2} z_2 \hat{S}^2 + \frac{1}{6} z_3 \hat{S}^3 \,, \qquad S_c = s_1 \hat{S} + \frac{1}{2} s_2 \hat{S}^2 + \frac{1}{6} s_3 \hat{S}^3 \,,$$

$$z_1 = \frac{r}{3} \qquad z_2 = \frac{r}{18}\left[3 - 8r + \frac{4r}{\xi} - 2\frac{r^2}{\xi^2}\right]$$

$$z_3 = \frac{r^2}{54}\left(\frac{6r^3}{\xi^4} + \frac{24r^2}{\xi^2} - \frac{4r^2}{\xi^3} - \frac{48r}{\xi} - \frac{15r}{\xi^2} + 64r + \frac{18}{\xi} - 36\right)$$

$$s_1 = (f_c - r) \qquad s_2 = \frac{1}{12}\left[3f_c(1-2f_c) - r\left(3 - 8r + \frac{4r}{\xi} - 2\frac{r^2}{\xi^2}\right)\right],$$

$$s_3 = -\frac{1}{2} f_c^2 (3 - 4f_c) - \frac{r^2}{18}\left(\frac{6r^3}{\xi^4} + \frac{24r^2}{\xi^2} - \frac{4r^2}{\xi^3} - \frac{48r}{\xi} - \frac{15r}{\xi^2} + 64r + \frac{18}{\xi} - 36\right)$$

$$f_A \equiv \frac{\gamma_{A\sigma}\Omega_\sigma}{\Omega_A + \gamma_{A\sigma}\Omega_\sigma} \qquad r \equiv \xi\, \tilde{r} \qquad \xi \equiv \frac{f_r}{\Omega_\sigma} \qquad \tilde{r} \equiv \frac{3\Omega_\sigma}{4 - \Omega_\sigma}$$