How Sensitive is the CMB to a Single Lens?

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arXiv:1105.2940: B. Rathuas, A. Fialkov and N. Itzhaki
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- Can they all be due to an anomalously large structure? Cruz et al, Turok and Spergel, Kovetz et al, Inoue and Silk, Fialkov et al
- Can we detect such a structure by means of WL?

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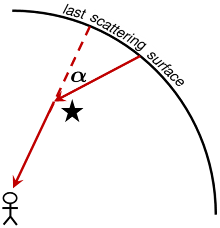

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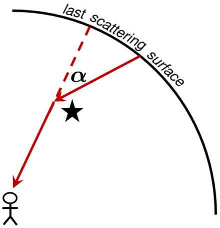

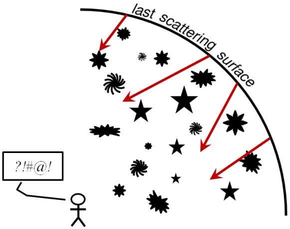

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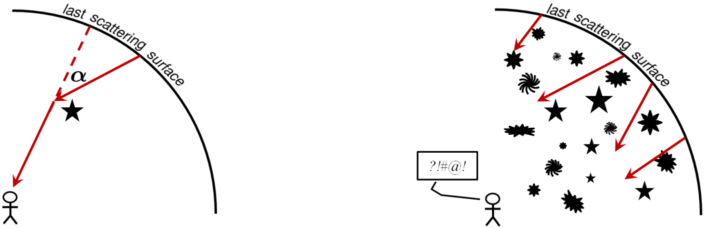

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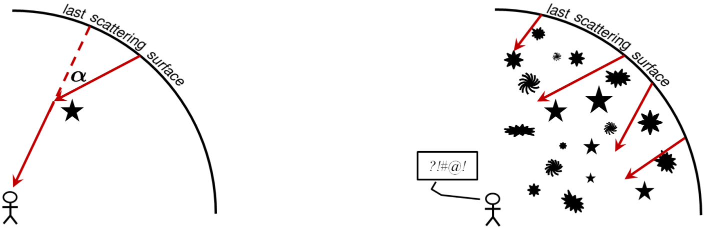

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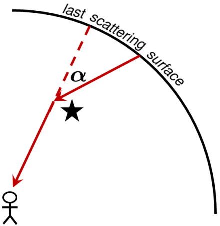

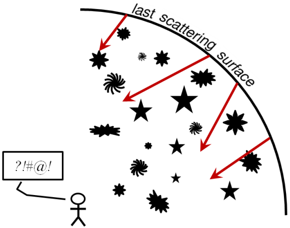

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- Now construct a S/N ratio.

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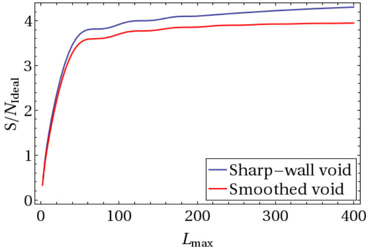
- An example: a void.



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- The S/N is then

$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)^2 = \frac{\epsilon^2}{2} \sum \frac{\left|C_{ij}^{(1)}\right|^2}{C_{ii}^{(0)} C_{jj}^{(0)}} = \int \frac{d\mathbf{l}}{(2\pi)^2} \frac{d\mathbf{l}'}{(2\pi)^2} \frac{|C^{(1)}(\mathbf{l},\mathbf{l}')|^2}{C(l)C(l')}.$$

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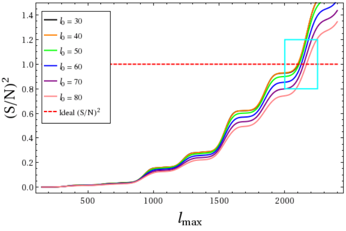
Ideal and Realistic S/N



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- Each leg:
  - 2D momentum
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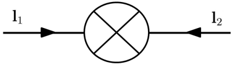
- Integrate loop mom.
- Single lens vertex



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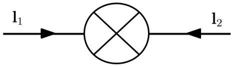

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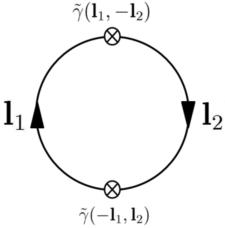

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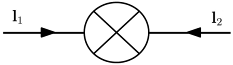

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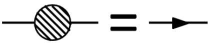

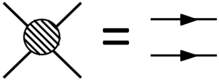

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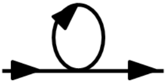

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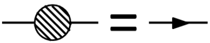

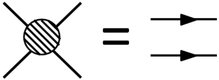

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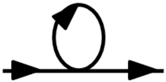

and this is indeed the case in ΛCDM WL.

How does this come about?

- First, write a 4-point function in terms of the primordial temperature field.
- Second, keep only second (lowest) order in the deflection potential

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- Finally, consider, for example, the following contraction

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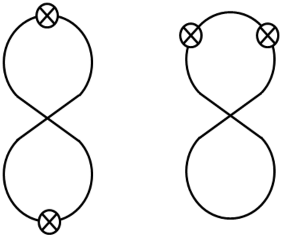
Resolving the puzzle

- 2-loop corrections to the S/N.



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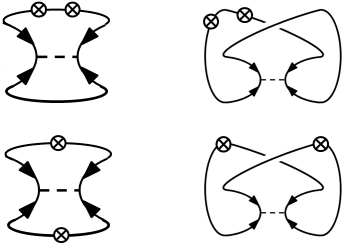

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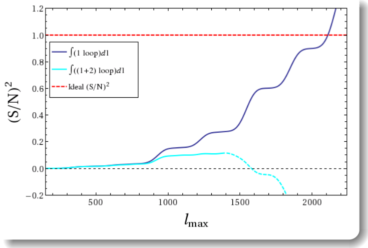

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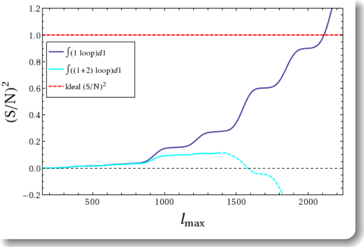

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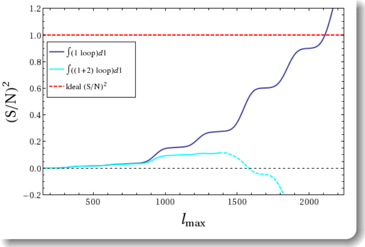

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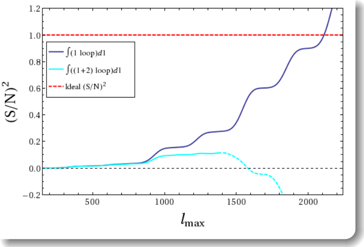

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Example

Cosmic void

- WMAP Cold-spot
- Argued: S/N ~ 40
- *But:*
  - $\mathrm{S/N}_{\text{Ideal}} \sim 4$
  - $\mathrm{S/N}_{\text{Acc.}} \sim 1$

Thank You!