A Conformal Bi-metric Model for the Inflationary Phase

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Outline

In the context of Modern Cosmology, we analyze the role of particular bi-metric Model as an effective action for the Inflationary phase

- MacDowell-Mansouri Action and our extension
- Cosmological Solution
- Black Hole Solution
- Liouville and Yang-Mills Theories
- Final Remarks

First order formalism

The first-order formalism takes as its fundamental variables a local frame (tetrad or vierbein) e_μ^a and a spin connection ω_μ^{ab} with

$$g_{\mu\nu} = \eta_{ab} e_\mu^a e_\nu^b \qquad \star F_{\mu\nu} = \frac{1}{2}\sqrt{-g}\,\varepsilon_{\alpha\beta\mu\nu} F^{\alpha\beta} \tag{1}$$

$$F = D\omega = d\omega + \omega \wedge \omega \tag{2}$$

$$T = De = de + \omega \wedge e \tag{3}$$

The tetrad can be viewed as a map from a fixed, four-dimensional space-time $\mathcal{M}$ to the tangent space $\mathcal{T}M$.

MacDowell and Mansouri formulated gravity as the gauge theory of $SO(4,1)$. The Lie algebra has a Killing orthogonal splitting:

$$\mathbf{so}(4,1) \cong \mathbf{so}(3,1) \oplus \mathbb{R}^{3,1} \tag{4}$$

$$A = \omega + \sqrt{\frac{\Lambda}{3}} e \tag{5}$$

$$F = dA + A \wedge A = \left(d\omega + \omega \wedge \omega - \frac{\Lambda}{3} e \wedge e \right) + De \tag{6}$$

$$S_{MM}[A] = -\frac{3}{16\pi G \Lambda} \int tr(F \wedge \star F) \tag{7}$$

Extended Gauge Gravity Model

Parameters: $M_P = \sqrt{\frac{3}{4\pi G}}$ $\quad M_I = \sqrt{\frac{\Lambda}{3}}$

Multi-connection: $\mathcal{A} = (A, i\overline{A})$

$$\mathcal{F} = \mathcal{D}\mathcal{A} = (D_\omega A, iD_{\overline{\omega}}\overline{A}) = (F, i\overline{F}) \qquad (8)$$

$$S = \frac{1}{12g^2}\int tr(\mathcal{F} \wedge \star\mathcal{F}) \qquad (9)$$

conformal "gauge fixing": $\overline{e} = \left(\frac{\phi}{M_P}\right) e$

$g = \left(\frac{M_I}{M_P}\right) < 1$

$$S = \frac{M_P^2}{12} \int d^4x \left[-\sqrt{-g}(R - 6M_I^2) + \sqrt{-\overline{g}}(\overline{R} - 6M_I^2) \right] =$$

$$\int d^4x \sqrt{-g} \left[-\frac{M_P^2}{12}(R - 6M_I^2) + \frac{1}{2}\partial^\mu \phi \partial_\mu \phi + \frac{1}{12} R\phi^2 - \frac{1}{2} \left(\frac{M_I}{M_P} \right)^2 \phi^4 \right]$$

with $\overline{g}_{\mu\nu} = \left(\frac{\phi}{M_P} \right)^2 g_{\mu\nu}$

$$\Box\phi - \frac{1}{6} R\phi + 2 \left(\frac{M_I}{M_P} \right)^2 \phi^3 = 0 \qquad (10)$$

$$\frac{M_P^2}{6} G_{\mu\nu} + \frac{1}{2}(M_P M_I)^2 g_{\mu\nu} = T^{\phi}_{\mu\nu} \qquad (11)$$

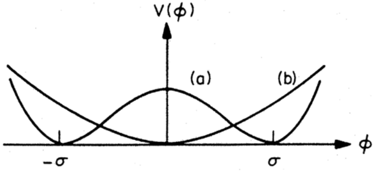
FRW metric

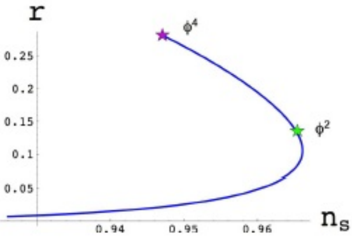
$$R = 12M_I^2 \qquad \rightarrow \qquad V(\phi) = \frac{1}{2}\left(M_I M_P\right)^2 \left[1 - \left(\frac{\phi}{M_P}\right)^2\right]^2 \quad (12)$$

In the context of Cosmological Standard Model:
$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

$$R = 6\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right] = 12M_I^2 \qquad \ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0$$

$$a(t) = a_0\sqrt{\cosh 2M_I t} \quad (13)$$





$N = 60$ (Kallosh-Linde '07, Boyanovsky et al. '09)

When $\phi \sim M_P \quad \rightarrow \quad S \sim 0$ (reheating)

Introduction

MTZ Black Hole solution:

$$ds^2 = -\left[-M_l^2 r^2 + \left(1 - \frac{GM}{r}\right)^2\right] dt^2 + \left[-M_l^2 r^2 + \left(1 - \frac{GM}{r}\right)^2\right]^{-1} dr^2 + r^2 d\Omega^2 \qquad (14)$$

where $0 \leqslant r < \infty$, $d\Omega^2$ is the metric of $\mathbb{S}^2$ and the scalar field is given by

$$\phi(r) = M_P^2 \frac{GM}{r - GM} \qquad (15)$$

The Black Holes have inner, event and cosmological horizons:

$$r_{-} = \frac{1}{2M_I}[-1 + \sqrt{1 + 4MM_I}] \tag{16}$$

$$r_{+} = \frac{1}{2M_I}[1 - \sqrt{1 - 4MM_I}] \tag{17}$$

$$r_{++} = \frac{1}{2M_I}[1 + \sqrt{1 - 4MM_I}] \tag{18}$$

We can calculate also the temperature (Winstanley '05):

$$T = \frac{M_I}{2\pi}\sqrt{1 - 4MM_I} \tag{19}$$

Properties:
1) The no-hair theorem is not valid with a non-minimally coupled scalar field
2) MTZ BHs are unstable BHs: This implies that under small perturbations the Black Hole would lose its hair (Hod '08) and decay in a stable BH.

The short lifetime of an unstable hairy Black Hole is compatible with the fact that Inflationary Phase is also a short period in the evolution of the Universe.

A PBH is formed not by the gravitational collapse of a large star but by the extreme density of matter present during the universe's early expansion.

MTZ BHs are Primordial BHs

$$S_D = \int d^D x \sqrt{-g} \left[\frac{1}{2} \partial^\mu \phi \partial_\mu \phi + f(\phi) R + U(\phi) \right] \tag{20}$$

$$f(\phi) \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + \frac{1}{2} \partial_\mu \phi \partial_\nu \phi - \frac{1}{4} g_{\mu\nu} \partial^\gamma \phi \partial_\gamma \phi - f'(\phi) \partial_\mu \partial_\nu \phi$$
$$- f''(\phi) \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} f'(\phi) \Box \phi + g_{\mu\nu} f''(\phi) \partial^\gamma \phi \partial_\gamma \phi - \frac{1}{2} g_{\mu\nu} U(\phi) = 0$$

$$\Box \phi - f'(\phi) R - U'(\phi) = 0 \tag{21}$$

imposing $R = const$ we have (G.P. '11)

$$f(\phi) = \frac{1}{4} \left(\frac{D-2}{D-1} \right) \frac{\phi^2}{2} + \beta \phi + \delta \tag{22}$$

$$U(\phi) = C\left[\left(\frac{D-2}{2D}\right)\phi + 2\left(\frac{D-1}{D}\right)\beta\right]^{\frac{2D}{D-2}} + \alpha^2\left[2\left(\frac{D-1}{D}\right)\beta^2 - \left(\frac{D-2}{D}\right)\delta\right] \quad (23)$$

$\beta = \frac{(4-D)(3-D)}{2}$

$\delta = -\frac{M_P^2}{12}$ $\qquad \alpha^2 = \frac{6D}{D-2}M_I^2$ $\qquad C = \frac{1}{2}\left(\frac{M_I}{M_P}\right)^2$

for $D = 2$, $f = \phi + \delta$ and there is the finite limit

$$\lim_{D\to 2}\left[\left(\frac{D-2}{2D}\right)\phi + \frac{(D-1)(4-D)(3-D)}{D}\right]^{\frac{2D}{D-2}} = e^{\phi} \quad (24)$$

we obtain

$$S_2 = \int d^2x\sqrt{-g}\left[\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \phi R + Ce^\phi - \frac{M_P^2}{12}R\right] \qquad (25)$$

1)Quantum version of Liouville action is fundamental in the string theory: conformal anomaly of Polyakov action
2)Important in the microscopic description of Hawking entropy (Solodukhin '03, Carlip '01)
3) AdS_3/CFT_2 (Henneaux '95)

Yang-Mills Theory and Gauge Gravity

Are there some correlations between our Extended Gauge Gravity Model and Yang-Mills Theory?
if we replace $\overline{F}$ with F_{YM}, we have

$$S = \frac{M_P^2}{12} \int d^4x \sqrt{-g} \left[-(R - 2\Lambda) + \frac{1}{4} tr\left(F_{\mu\nu} F^{\mu\nu}\right) \right] \qquad (26)$$

- the same cosmological solution because the trace of stress-energy tensor is equal to zero
- the MTZ Black Hole metric coincides with the extremal Reissner-Nordstrom-de Sitter Black Hole metric (Q=M)

Final Remarks

We have introduced a minimal Gauge Gravity Extension of General Relativity.

Is it only Mathematics or our ϕ can have a physical meaning?

Cosmological solution: compatible with the accelerated expansion and the spectral index is good agreement with its experimental value.

Black Hole solution: compatible with the theorized existence of Primordial Black Holes in the Early Universe.

Relation with more fundamental theories: Lioville Theory in two dimensions and Yang-Mills theory in four dimensions.

In our approach Inflationary Theory can seen as a pure geometric theory.