UV-protected Natural Inflation: Primordial Fluctuations and non-Gaussian Features

Yuki Watanabe

Arnold Sommerfeld Center, Ludwig-Maximilians-University, Munich

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Slow Roll Inflation

A scalar field ϕ is a good candidate of an Inflaton as

$$\rho = \frac{1}{2}\dot{\phi}^2 + V \ , p = \frac{1}{2}\dot{\phi}^2 - V$$

By geometrical identity (Raychaudhuri eq.)

$$\ddot{a} \propto -(\rho + 3p) \propto -(\dot{\phi}^2 - V)$$

$$\Downarrow$$

$$\begin{array}{lcl} \dot{\phi}^2 & \ll & V \ , \text{ Inflation happens (“slow roll”)} \\ \dot{\phi}^2 & \sim & V \ , \text{ Inflation ends} \end{array}$$

Q: How do we achieve the slow roll for sufficient time?

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$$\dot{\phi}^2 \ll V \ , \text{ Inflation happens (“slow roll”)}$$
$$\dot{\phi}^2 \sim V \ , \text{ Inflation ends}$$

Q: How do we achieve the slow roll for sufficient time?

(1) Let V have a non-trivial (positive) minimum,
(2) increase friction, or (3) fine-tune V to be flat.

Increasing friction

Let us ignore the gravity for a moment. Our goal is to achieve

$$E \simeq V,\ \tilde{\epsilon} \equiv -\frac{\dot{E}}{E^2} \ll 1 \text{ and } \tilde{\delta} \equiv \frac{\dot{\tilde{\epsilon}}}{\epsilon E} \ll 1$$

in a non-equilibrium point of V for a long time.

The friction must dominate over the acceleration:

$$\tilde{\mu}\dot{\phi} \simeq -V'$$

In order to have an almost constant energy, the friction coefficient must also be roughly constant. In this case,

$$\tilde{\epsilon} \simeq \frac{V'^2}{V^2}\frac{1}{\tilde{\mu}} \text{ and } \tilde{\delta} \simeq -2\frac{V''}{V}\frac{1}{\tilde{\mu}} + 2\tilde{\epsilon}$$

Slow roll is achieved if $\tilde{\mu}$ is large and roughly constant.

Increasing friction II

There are two ways to implement the friction:

$$(1)\ \ddot{\phi} + \tilde{\mu}\dot{\phi} = -V' \text{ and}$$

$$(2)\ \mu\left(\ddot{\phi} + 3E\dot{\phi}\right) = -V' \text{ with } \tilde{\mu} = 3E\mu$$

The friction dominates over the acceleration if

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As a result,

(1) $\tilde{\tilde{\delta}} \sim \tilde{\delta}(E/\tilde{\mu})$ is a new parameter controlling the system if $\tilde{\mu} \not\propto E$. $\Rightarrow$ New d.o.f.!

(2) $\tilde{\tilde{\delta}} \sim \tilde{\delta} \Rightarrow$ Good starting point to avoid new d.o.f.

Increasing friction with gravity

Let us now introduce gravity. Since the gravitational Hamiltonian density in the FRW Univ. is $\mathcal{H} = 3M_p^2 H^2$, we may identify

$$V \sim \mathcal{H},\ E \sim H.$$

The slow roll is then achieved by the Friedmann eqn

$3M_p^2 H^2 \simeq V$ and $\tilde{\mu} = 3H\mu(H)$ if no new d.o.f. is added.

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A typical enhancement of friction could be

$$\mu(H) = 1 + \frac{H^2}{M^2}.$$

$$\mu\left(\ddot{\phi} + 3H\dot{\phi}\right) = -V' \Rightarrow t_{\text{eff}} \simeq \frac{t}{\sqrt{\mu}} \text{ as } -\frac{\dot{\mu}}{\mu H} \ll 1.$$

If $H^2 \gg M^2$ during inflation, scalar field's clock is moving slower than observer's clock and friction is enhanced!

Gravitationally Enhanced Friction (GEF): Realization

In order to realize the enhanced friction in a covariant manner, we promote the rescaling to all coords.

$$\partial_\mu \to \sqrt{\mu}\partial_\mu, \ \mu = 1 + \frac{H^2}{M^2}$$

By noticing that during slow roll, $G^{\mu\nu} \simeq -3H^2 g^{\mu\nu}$.

The enhanced friction is covariantly realized by shifting the kinetic action

$$\mathcal{L} \sim g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \to \left(g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2}\right)\partial_\mu\phi\partial_\nu\phi$$

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Thanks to Bianchi identities, this action has the properties:

- EoM of ϕ is shift and *curved* Galilean invariant [Germani et al 2011]: $\phi \to \phi + c + c_\alpha \int_{\gamma,x_0}^{x} \xi^\alpha$
- Propagates only spin 0 (scalar) and spin 2 (graviton) particles (no higher derivatives, Lapse and Shift are still Lagrange multipliers)
- Makes harder for a scalar field to roll down its own potential!

Full action of the GEF inflation

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}\Delta^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V\right],$$

$$\text{where } \Delta^{\alpha\beta} \equiv g^{\alpha\beta} - \frac{G^{\alpha\beta}}{M^2}.$$

In a FRW background, the Friedmann and field eqs read

$$H^2 = \frac{1}{3M_p^2}\left[\frac{\dot\phi^2}{2}\left(1+9\frac{H^2}{M^2}\right) + V\right], \quad \partial_t\left[a^3\dot\phi\left(1+3\frac{H^2}{M^2}\right)\right] = -a^3V' .$$

During slow roll in the high friction limit ($H^2/M^2 \gg 1$), the eqs are simplified as

$$H^2 \simeq \frac{V}{3M_p^2} , \quad \dot\phi \simeq -\frac{V'}{3H}\frac{M^2}{3H^2} .$$

Power of the GEF mechanism

Consistency of the eqs requires the slow roll parameters to be small, i.e.

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1 \ , \quad \delta \equiv \frac{\ddot{\phi}}{H\dot{\phi}} \ll 1 \ .$$

By explicit calculation, one can show that

$$\epsilon \simeq \frac{V'^2 M_p^2}{2V^2}\frac{M^2}{3H^2}, \quad \delta \simeq -\frac{V'' M_p^2}{V}\frac{M^2}{3H^2} + 3\epsilon = -\eta + 3\epsilon \ , \quad \eta \equiv \frac{V'' M_p^2}{V}\frac{M^2}{3H^2}.$$

We see that, no matter how big the slow roll parameters of GR are

$$\epsilon_{GR} \equiv \frac{V'^2 M_p^2}{2V^2} \qquad \text{and} \qquad \eta_{GR} \equiv \frac{V'' M_p^2}{V},$$

there is always a choice of scale $M^2 \ll 3H^2$, during inflation, such that slow roll parameters are small.

Cosmological perturbations in the GEF inflation

ADM form

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)^2$$

- Use the gauge $\delta\phi = 0$
- then: $h_{ij} = a^2[(1 + 2 \underbrace{\zeta}_{\text{curvature perturbation}})\delta_{ij} + \underbrace{\gamma_{ij}}_{\text{gravitational waves}}]$
- Vary wrt the constraints N, N^i, substitute back into the action and canonically normalize ζ and γ_{ij}
- $N = 1 + \frac{\Gamma}{H}\dot{\zeta},\ N^i = -\frac{\Gamma}{H}\partial_i\zeta + \frac{\Sigma}{H^2}\partial_i\partial^{-2}\dot{\zeta}$
- $\Gamma(\dot{\phi}, H, M) \simeq 1 + \frac{2}{3}\epsilon,\ \Sigma(\dot{\phi}, H, M) \simeq \frac{\dot{\phi}^2}{2M_p^2}\left[1 + \frac{3H^2}{M^2}\right] \simeq \epsilon H^2$ in the high friction limit $H \gg M$.

Curvature perturbation spectrum

- $\mathcal{L}_{\zeta^2} = \frac{1}{2}[v'^2 - c_s^2(\partial_i v)^2 + \frac{z''}{z} v^2]$ with $c_s^2 = 1 - \mathcal{O}(\epsilon)$
- $\langle \hat{\zeta}_k \hat{\zeta}_{k'} \rangle = (2\pi)^3 \delta^{(3)}(k + k') \frac{2\pi^2}{k^3} \mathcal{P}_\zeta$ where $\mathcal{P}_\zeta = \frac{H^2}{8\pi^2 \epsilon c_s M_p^2}$
- spectral index: $n_s - 1 = \frac{d \ln \mathcal{P}_\zeta}{d \ln k} \approx -2\epsilon - 2\delta$
- running of the spectral index: $\frac{dn_s}{d \ln k} \approx -6\epsilon\delta - 2\delta\delta' + 2\delta^2$

Matching with the WMAP data, $\mathcal{P}_\zeta = 2 \times 10^{-9}$, we get a relation

$$\frac{M^2}{H^2} = \frac{10^9}{8\pi^2} \frac{V^3}{V'^2 M_p^6}$$

Note that scalar perturbations are slightly sub-luminal.
Can this lead to observational consequences?
(Any GW or NG due to the new non-linear interaction?)

Gravitational wave spectrum

- $\mathcal{L}_{\gamma^2} = \sum_{\lambda=\pm 2} \frac{1}{2}[v_t'^2 - c_{gw}^2(\partial_k v_t)^2 + \frac{z_t''}{z_t} v_t^2]$ with $c_{gw}^2 = 1 + \mathcal{O}(\epsilon)$
- $\langle \hat{\gamma}_k \hat{\gamma}_{k'} \rangle = (2\pi)^3 \delta^{(3)}(k + k') \frac{2\pi^2}{k^3} \mathcal{P}_\gamma$ where $\mathcal{P}_\gamma = \frac{2H^2}{\pi^2 c_{gw}(1+\epsilon/3) M_p^2}$
- spectral index is red: $n_t = \frac{d \ln \mathcal{P}_\gamma}{d \ln k} \approx -2\epsilon$
- tensor to scalar ratio: $r = \frac{\mathcal{P}_\gamma}{\mathcal{P}_\zeta} = 16\epsilon = -8 n_t$

Note that GWs are slightly "super-luminal", but this does not mean "acausal" since the causal structure is set by the propagation of GWs.

GEF saves $\lambda\phi^4$ model

The $\lambda\phi^4$ model predicts a red spectrum:

$$V = \frac{\lambda}{4}\phi^4, \quad n_s - 1 \simeq -\frac{40}{3}\frac{M^2}{H^2}\frac{M_p^2}{\phi_i^2} \simeq -5\epsilon$$

$$N_e = \frac{5}{3(1-n_s)}.$$

For $n_s - 1 = -0.03$, one obtains

$\epsilon \simeq 0.0167$, $N_e \simeq 56$, $r \simeq 0.1$ and

$$\frac{\phi_i}{M_p} \simeq 7\times 10^{-2}\left(\frac{0.1}{\lambda}\right)^{1/4}, \quad \frac{H}{M_p} \simeq 5\times 10^{-5}, \quad \frac{M}{M_p} \simeq 2\times 10^{-7}\left(\frac{0.1}{\lambda}\right)^{1/4},$$

The values are compatible with the WMAP data.

Non-Gaussianity in the GEF inflation

- We compute the non-Gaussian feature of the scalar fluctuations from the cubic action.
- We use the comoving gauge variable, ζ, since it is conserved outside the horizon (at least at order ϵ).
- The leading order effect appears in the bispectrum or the three-point function. Since $\mathcal{L}^{GEF}_{\zeta^3} \sim \mathcal{O}(\epsilon^2)$, we get $f_{NL} \sim \langle\zeta^3\rangle/\langle\zeta^2\rangle^2 \sim \mathcal{O}(\epsilon)$.

Bispectrum: three-point correlation function

As in the power spectrum, the bispectrum of ζ is defined by the three-point correlation function:

$$\langle\hat{\zeta}(\tau,\mathbf{k}_1)\hat{\zeta}(\tau,\mathbf{k}_2)\hat{\zeta}(\tau,\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\zeta(k_1,k_2,k_3).$$

One can evaluate the three-point correlator by using the in-in formalism [Maldacena 2002, Weinberg 2005]. In the lowest order,

$$\begin{aligned}&\langle\hat{\zeta}(0,\mathbf{k}_1)\hat{\zeta}(0,\mathbf{k}_2)\hat{\zeta}(0,\mathbf{k}_3)\rangle\\ &\quad = -i\int_{-\infty}^{0} d\tau a\langle 0|[\hat{\zeta}(0,\mathbf{k}_1)\hat{\zeta}(0,\mathbf{k}_2)\hat{\zeta}(0,\mathbf{k}_3),\hat{H}_{int}(\tau)]|0\rangle,\end{aligned}$$

where we have set the initial and final times as $\tau_i=-\infty$ and $\tau_f=0$, respectively. The interaction Hamiltonian is given by

$$\hat{H}_{int}(\tau) = -\int d^3x\hat{\mathcal{L}}^{GEF}_{\zeta^3}.$$

The 3-pt fn can be calculated from each term of the int Hamiltonian:

- $H_{int}^{(1)}(\tau) = -c_1 a^3 \int d^3x \zeta \dot{\zeta}^2$

  $$B_\zeta^{(1)} = \frac{c_1 H^4}{16\epsilon_s^3 M_p^6} \frac{1}{(k_1 k_2 k_3)^3} \left(\frac{k_2^2 k_3^2}{K} + \frac{k_1 k_2^2 k_3^2}{K^2} + \mathrm{sym} \right),$$

- $H_{int}^{(2)}(\tau) = -c_2 a^3 \int d^3x \dot{\zeta}^3$

  $$B_\zeta^{(2)} = \frac{3 c_2 H^5}{8\epsilon_s^3 M_p^6} \frac{1}{k_1 k_2 k_3 K^3},$$

- $H_{int}^{(3)}(\tau) = -c_3 a \int d^3x \partial_i^2 \zeta \dot{\zeta}^2$

  $$B_\zeta^{(3)} = \frac{3 c_3 H^6}{4\epsilon_s^3 c_s^2 M_p^6} \frac{1}{k_1 k_2 k_3 K^3},$$

where $k = |\mathbf{k}|$, $K = k_1 + k_2 + k_3$ and "sym" denotes the symmetric terms with respect to $k_1,\ k_2,\ k_3$.

- $H_{int}^{(4)}(\tau) = -c_4 a \int d^3x \zeta (\partial_i \zeta)^2$

$$B_\zeta^{(4)} = \frac{c_4 H^4}{16\epsilon_s^3 c_s^2 M_p^6} \frac{1}{(k_1 k_2 k_3)^3} \times \left[(\mathbf{k}_1 \cdot \mathbf{k}_2 + \mathbf{k}_2 \cdot \mathbf{k}_3 + \mathbf{k}_3 \cdot \mathbf{k}_1) \left(-K + \frac{k_1 k_2 + k_2 k_3 + k_3 k_1}{K} + \frac{k_1 k_2 k_3}{K^2} \right) \right],$$

- $H_{int}^{(5)}(\tau) = -c_5 a \int d^3x \dot{\zeta} (\partial_i \zeta)^2$

$$B_\zeta^{(5)} = \frac{c_5 H^5}{32\epsilon_s^3 c_s^2 M_p^6} \frac{1}{(k_1 k_2 k_3)^3} \left[\frac{k_1^2 (\mathbf{k}_2 \cdot \mathbf{k}_3)}{K} \left(1 + \frac{k_2 + k_3}{K} + \frac{2 k_2 k_3}{K^2} \right) + \mathrm{sym} \right],$$

- $H_{int}^{(6)}(\tau) = -c_6 a \int d^3x \dot{\zeta} \partial_i \zeta \partial_i \chi$

$$B_\zeta^{(6)} = \frac{c_6 H^4}{32\epsilon_s^2 c_s^2 M_p^6} \frac{1}{(k_1 k_2 k_3)^3} \left[\frac{(\mathbf{k}_1 \cdot \mathbf{k}_2) k_3^2}{K} \left(2 + \frac{k_1 + k_2}{K} \right) + \mathrm{sym} \right],$$

- $H_{int}^{(7)}(\tau) = -c_7 a \int d^3x \dot{\zeta}^2 \partial_i^2 \chi$

$$B_\zeta^{(7)} = \frac{3\tilde{c}_7 H^5}{8\epsilon_s^3 M_p^6} \frac{1}{k_1 k_2 k_3 K^3}, \quad \tilde{c}_7 \equiv \epsilon c_7 .$$

By using the Wick's theorem, we obtain the contribution from field redefinition $\zeta \to \zeta + (\epsilon/2 + \delta/2)\zeta^2$:

$$B_\zeta^{\rm redef}(k_1, k_2, k_3) = \frac{(\epsilon+\delta)H^4}{16\epsilon_s^2 c_s^2 M_p^4}\left(\frac{1}{k_1^3 k_2^3} + \frac{1}{k_2^3 k_3^3} + \frac{1}{k_3^3 k_1^3}\right).$$

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In the squeezed limit,

$$B_\zeta^{(1)}(k_1, k_2 \to k_1, k_3 \to 0) = \frac{3\epsilon}{2} P_\zeta(k_1) P_\zeta(k_3),$$

$$B_\zeta^{(4)}(k_1, k_2 \to k_1, k_3 \to 0) = -\frac{3\epsilon}{2} P_\zeta(k_1) P_\zeta(k_3),$$

$$B_\zeta^{\rm redef}(k_1, k_2 \to k_1, k_3 \to 0) = 2(\epsilon + \delta) P_\zeta(k_1) P_\zeta(k_3).$$

Other terms are sub-dominant in this limit, and thus we get the consistency relation:

$$\frac{12}{5} f_{NL} = \frac{B_\zeta(k_1, k_2 \to k_1, k_3 \to 0)}{P_\zeta(k_1) P_\zeta(k_3)} = 1 - n_s$$

Natural Inflation

In natural inflation, the field ϕ is a pseudo-Nambu-Goldstone Boson with decay constant f and periodicity $2\pi f$ [Freese et al 1990].

$$\mathcal{L} = \sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - me^{i\frac{\phi}{f}}\bar{\psi}(1+\gamma_5)\psi - \bar{\psi}\not{D}\psi - \frac{1}{2}\mathrm{Tr}F_{\alpha\beta}F^{\alpha\beta}\right],$$

- where ψ is a fermion charged under the (non-abelian) gauge field with field strength $F_{\alpha\beta}$, $\not{D} = \gamma^\alpha \mathcal{D}_\alpha$ is the gauge invariant derivative and $m \sim f$ is the fermion mass scale after spontaneous symmetry breaking.
- The action is invariant under the chiral (global) symmetry $\psi \to e^{i\gamma_5\alpha/2}\psi$, where α is a constant.
- This symmetry is related to the invariance under shift symmetry of ϕ, i.e. $\phi \to \phi - \alpha\ f$.

Natural Inflation II

- Suppose the chiral symmetry is broken at energies $f >$TeV (like in the QCD axion case) [chiral anomaly induces $(\phi/f)F\tilde{F}$]
- a potential of pNGB ϕ is produced by the instanton effect:

$$V(\phi) \sim \Lambda^4 \left[1 \pm \cos\frac{\phi}{f}\right]$$

which is protected from QG UV corrections by the restoration of global shift symmetry $\phi \to \phi + c$ as $\Lambda/M_p \to 0$

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- If $\Lambda \sim 10^{16}$ GeV (GUT scale), inflation is produced with

$$n_s - 1\,(\propto \epsilon) \simeq -\frac{M_p^2}{8\pi f^2}$$

- so $n_s - 1 \simeq -0.04 \to f > M_p$
  $\Rightarrow$ The potential may not be protected from QG UV corrections.

GEF saves Natural Inflation

Once again we can increase the friction so that

$$\epsilon \to \frac{\epsilon_{old}}{\mu} \Rightarrow n_s - 1 \sim -\frac{M_p^2}{8\pi f \mu}$$

For large enough friction μ, we have $f \ll M_p$

All the coupling scales are sub-Plankian!
(i.e. no UV modifications of the potential)

The new coupling $G^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$ is the unique that

- *does not* introduce any new d.o.f.
- *is invariant* under the global unbroken symmetry $\phi \to \phi + c$.

UV-protected Natural Inflation

Inspired by Natural Inflation, we will consider the following tree-level Lagrangian for a single pseudo-scalar field ϕ [Germani & Kehagias 2010]

$$\mathcal{L} = \sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}\Delta^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - me^{i\frac{\phi}{f}}\bar{\psi}(1+\gamma_5)\psi - \bar{\psi}\not{\mathcal{D}}\psi - \frac{1}{2}\mathrm{Tr}F_{\alpha\beta}F^{\alpha\beta}\right],$$

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Red spectrum from UV-protected Inflation

Small field branch:

$$V(\phi) \simeq \Lambda^4 \left(2 - \frac{\phi^2}{2f^2}\right)$$

- $n_s - 1 \simeq -\frac{1}{3}\frac{M^2}{H^2}\frac{M_p^2}{f^2} < 0 \Rightarrow$ Red spectrum!

Matching with $\mathcal{P}_\zeta = 2 \times 10^{-9}$ and $1 - n_s = 0.04$,

$$\frac{\Lambda^2}{M_p^2} = \frac{\pi\sqrt{6}}{10^5\sqrt{5}}\frac{\phi_i}{f}, \qquad \frac{M}{H} = \frac{\sqrt{3}}{5}\frac{f}{M_p}$$

- Consistent with the theoretical hierarchies of scales to protect the potential:

$$M \ll M\frac{M_p^2}{\Lambda^2} \ll f \ll M_p$$

- GW signal is small. Large field branch?

Red spectrum from UV-protected Inflation II

Large field branch:

$$V(\phi) \simeq \frac{1}{2} m^2 \phi^2 \ , \quad m \equiv \frac{\Lambda^2}{f}$$

- $n_s - 1 = -\frac{3}{2N_e} < 0 \Rightarrow 1 - n_s = 0.03$ for $N_e = 50$.

Matching with $\mathcal{P}_\zeta = 2 \times 10^{-9}$ and $1 - n_s = 0.03$,

$$\frac{\phi_i}{M_p} = \frac{2\pi\sqrt{6}}{\sqrt{5}} \times 10^{-5} \frac{M_p}{m} = \frac{1}{\pi}\sqrt{\frac{5}{3}} \times 10^{6} \frac{M}{M_p}$$

- Consistent with the theoretical hierarchies of scales to protect the potential:

$$M_p \sqrt{\frac{M}{m}} \ll \phi \ll f \ll M_p \text{ and } \Lambda \ll M_p$$

- GW signal is potentially detectable: $r \simeq 16\epsilon \simeq 0.08$

Non-Gaussianity from UV-protected Inflation

NG can be generated by the inverse decays of gauge fields if the inflaton is identified as a pseudo-scalar [Barnaby & Peloso 2010]:

$$f_{NL}^{\rm equil} \simeq 4.4 \times 10^{10} \mathcal{P}_\zeta^3 \frac{e^{6\pi\xi_i}}{\xi_i^9}, \quad \xi_i \equiv \frac{\dot{\phi}}{2f_i H} = \xi \frac{f}{f_i}, \quad \xi \equiv \frac{\dot{\phi}}{2fH}.$$

At sufficiently large $\xi_i \gtrsim \mathcal{O}(1)$, $f_{NL}^{\rm equil} \simeq 8400$, which excludes axion-like inflation models by the observations.

In the small field branch of the UV-protected inflation,

$$\xi \equiv \frac{\dot{\phi}}{2fH} \simeq \sqrt{\frac{\epsilon}{6}} \frac{M}{H} \frac{M_p}{f} = \frac{\sqrt{\epsilon}}{5\sqrt{2}} \simeq 2 \times 10^{-2} \ll 1.$$

No NG is generated by the gauge field that produces the inflaton potential, but we also have

$$\xi_i \simeq 2 \times 10^{-2} \ \frac{f}{f_i},$$

which allows a detactable signal for $f_i \sim 10^{-2} f$.

Conclusions

- By increasing the friction, inflation can be obtained with the SM Higgs or a pNGB.
- The friction can be enhanced by nonminimally coupling the Einstein tensor to the kinetic term of the Inflaton.
- This coupling is unique: *does not increase* no. of propagating d.o.f.
- Consistent with WMAP 7-years result.
- Non-Gaussianities from single-field inflation models with GEF are generically small.
- The parity violating interactions may produce detectable NG.