Cosmological decoherence in the framework of stochastic inflation

Jan Weenink

Institute for Theoretical Physics, Utrecht University & Nikhef

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Goal of this project:

"Perform a *quantitative* analysis of decoherence during inflation for realistic inflationary models (in this case a $\lambda\phi^4$ theory), in order to find out if it has an observable effect on the CMB."

In this talk I will

- show how to quantitatively describe decoherence,
- apply this to cosmological decoherence and discuss the difficulties,
- argue why stochastic inflation may help to overcome these difficulties.

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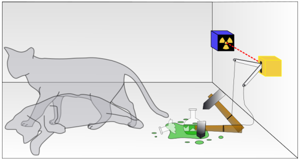
Decoherence

How does a quantum system become classical?

→ Measurement: "collapse of the wavefunction"



Decoherence (Zeh 1970) :

describes how a system S evolves into a state which *most closely resembles* a classical state

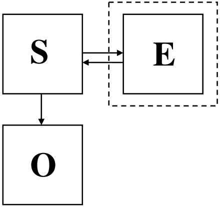
→ Need an irreversible interaction with an environment E

Decoherence and entropy

Consider a system S and an environment E which interact.

The total von Neumann entropy is conserved

$$S_{\text{vN}} = S_S + S_E + S_{SE} \quad \longrightarrow \quad \frac{dS_{\text{vN}}}{dt} = 0$$



Observer O observes only the system

$\rightarrow$ no knowledge about environment

$$S_{\text{vN}}^{\text{red}} = S_S \quad \longrightarrow \quad \frac{dS_{\text{vN}}^{\text{red}}}{dt} = \frac{dS_S}{dt} \neq 0$$

Loss of information leads to growth of entropy for system S

Decoherence and entropy 2

Decoherence can be due to:

- observationally inaccessible environment, *e.g.* heat bath
- observationally inaccessible correlators, *i.e.* n-point functions, $n \geq 3$
  Campo, Parentani 2008; Giraud, Serrau 2010; Koksma, Prokopec, Schmidt 2010

Suppose we can observe *only Gaussian correlators*:

Gaussian von Neumann entropy for quantum field ϕ:

$$S_g = \int \frac{d^3\vec{k}}{(2\pi)^3}\left[\frac{\Delta_k+1}{2}\ln\left(\frac{\Delta_k+1}{2}\right) - \frac{\Delta_k-1}{2}\ln\left(\frac{\Delta_k-1}{2}\right)\right]$$

Phase space area Δ:

$$\Delta_k^2 = 4\left[\langle\Delta\phi_k^2(t)\rangle\langle\Delta\pi_k^2(t)\rangle - \langle\frac{1}{2}\{\Delta\phi_k(t),\Delta\pi_k(t)\}\rangle^2\right]$$

For free theory, $\Delta = 1$ and $S = 0$; For interacting theory $\Delta > 1$ and $S > 0$

Allows for a *quantitative* description of decoherence

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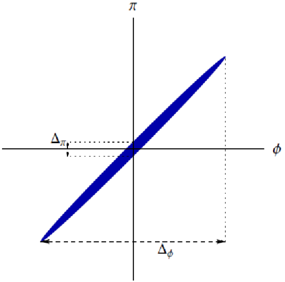
Phase space area

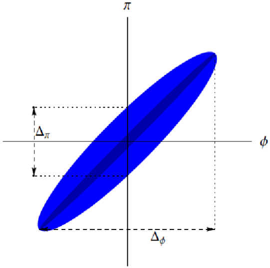
(Squeezed) initial state, $\Delta = 1$



interactions

$\longrightarrow$

Decohered state, $\Delta > 1$



For interacting theory, phase space area grows, $\Delta > 1$

Decoherence is reduced to calculating the growth of the phase space area

Cosmological decoherence

Quantum-to-classical transition during inflation:

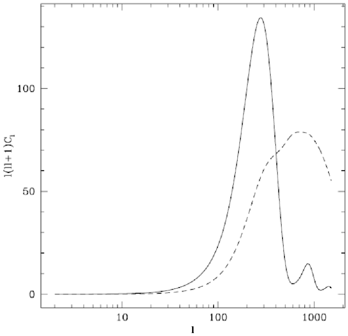
- Phase space area of inflaton field becomes extremely squeezed
- "Decoherence without decoherence" Polarski, Starobinsky 1994

But in general: interactions of inflaton field with itself or other fields

- Lead to growth of phase space area
- May be observable in the CMB $\longrightarrow$

Solid: no decoherence; Dashed: complete decoherence

"A realistic model will always lay between these two extremes"



Durrer, Sakellariadou 1997

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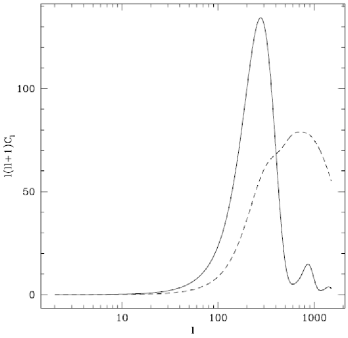

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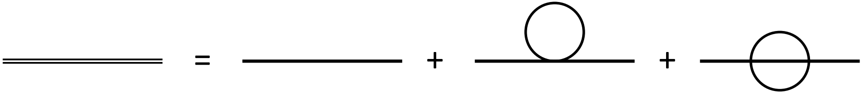
Quantitative calculation of cosmological decoherence

- Effect of cosmological decoherence on CMB depends on model
- We want to do a quantitative calculation for inflaton potential $V = \frac{\lambda}{4!}\phi^4$

Decoherence: growth of phase space area for self-interacting inflaton field

$\rightarrow$ Only have to find Gaussian (2-point) correlators

Challenging: in principle need to calculate loop effects in (quasi) de Sitter space



Late time behavior of correlators

Problem: free two point correlator in de Sitter space:

$$\langle \phi^2 \rangle = \left(\frac{H}{2\pi} \right)^2 \ln a + \ldots$$

- Secular behavior: higher loop terms scale as $(\ln a)^n$
- At late times, loop expansion becomes non-perturbative
- How to deal with this?

$\longrightarrow$ Need to resum the $\ln a$ terms

Guaranteed correct, but very challenging: *the 2PI effective action approach*

Derive nonlinear Kadanoff-Baym equations for the fully resummed propagators

Can we somehow find the correct late time behavior in a simpler way?

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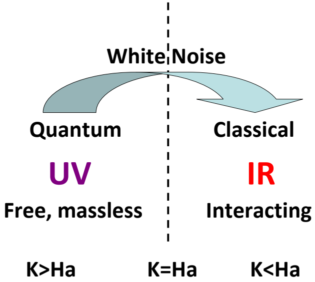
Stochastic inflation

Stochastic inflation Starobinsky 1985

- based on a separation between UV (sub-Hubble) modes and IR (super-Hubble) modes
- UV modes free and massless
- integrate out the fast (UV) modes
- sub-Hubble modes acts as white noise to super-Hubble modes
- IR modes treated classically



Stochastic inflation

Starobinsky equation for IR field ϕ:

$$\dot{\phi}(t,\mathbf{x}) = -\frac{1}{3H} V'\left(\phi(t,\mathbf{x})\right) + F(t,\mathbf{x})$$

Classical white noise:

$$\langle F(t,\mathbf{x}) F(t',\mathbf{x}) \rangle = \frac{H^3}{4\pi^2}\delta(t-t'),$$

Classical Langevin equation:

- Reproduces leading logs at each order in perturbation theory Tsamis, Woodard 2005
- Fully resummed field amplitudes → non-perturbative! Starobinsky, Yokoyama 1994

Could we use the framework of stochastic inflation to calculate the phase space area?

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Phase space approach to stochastic inflation

Hamilton equations in FLRW universe:

$$\dot{\Phi} = \frac{1}{a^3}\Pi_\Phi$$

$$\dot{\Pi}_\Phi = a\nabla^2\Phi - a^3\frac{\lambda}{3!}\Phi^3$$

Now separation between super- and sub-Hubble modes (Habib 1992):

$$\Phi = \phi + \phi_{\mathrm{UV}}$$

$$\Pi = \pi_\phi + \pi_{\mathrm{UV}}$$

Important assumption: UV field does not appear in potential V

- ϕ_{UV} and π_{UV} are free fields
- can integrate out

Stochastic Hamilton equations

Stochastic Hamilton equations (Weenink, Prokopec 2011):

$$\dot{\phi} = \frac{\pi_\phi}{a^3} + F_1(t, \mathbf{x})$$
$$\dot{\pi}_\phi = a\nabla^2\phi + F_2(t, \mathbf{x}) - a^3\frac{\lambda}{3!}\phi^3$$

Noise correlators satisfy

$$\langle F_i(t, \mathbf{k}) F_j^*(t', \mathbf{k})\rangle = f_{ij}\delta(k - \epsilon aH)(\epsilon aH)^{\cdot}\delta(t - t')$$

For a free field ($\lambda = 0$), the phase space area is:

$$\Delta_k^2 = 4\left[\langle\Delta\phi_k^2(t)\rangle\langle\Delta\pi_k^2(t)\rangle - \langle\frac{1}{2}\{\Delta\phi_k(t), \Delta\pi_k(t)\}\rangle^2\right]$$
$$= 4\theta(\epsilon aH - k)\left[f_{11}f_{22} - \left(\frac{f_{12} + f_{21}}{2}\right)^2\right] = \theta(\epsilon aH - k)$$

No decoherence for the *free* IR field ϕ! Habib 1992

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Solving the stochastic Hamilton equations

Now want to solve the stochastic Hamilton equations:

$$\dot{\phi} = \frac{\pi_\phi}{a^3} + F_1(t, \mathbf{x})$$

$$\dot{\pi}_\phi = a\nabla^2\phi + F_2(t, \mathbf{x}) - a^3\frac{\lambda}{3!}\phi^3$$

Treat ϕ and π_ϕ classically: makes sense when correlators that involve anticommutator $\gg$ correlators that involve commutator $\rightarrow$ true for super-Hubble field ϕ

(see also Tolley, Wyman 2008 for discussion about classicality)

Replace quantum noise terms by classical random variables

$\rightarrow$ drawn from Gaussian distributions with width $\sim f_{ij}$

$\rightarrow$ guarantees that $\Delta_k(t_{\rm init}) = 1$

$\rightarrow$ allows for numerical solution of growth of Δ (work in progress..)

Questions regarding the approach

Numerical solution of the stochastic Hamilton equations may provide the correct late time behavior of the phase space area.

Main questions:

- Can the stochastic Hamilton equations be treated classically?
- Important assumption: UV modes treated as free, massless field
  $\rightarrow$ Do UV-IR interactions give the dominant contribution to decoherence?

Validity of approach can be tested by comparing with a full 2PI calculation!

Summary and conclusions

Cosmological decoherence

- Approach to decoherence based on neglecting observationally inaccessible correlators
- Useful for a quantitative study of cosmological decoherence
- Loop calculations in de Sitter: secular terms, resummation $\rightarrow$ stochastic inflation

Phase space approach to stochastic inflation:

- Formulation stochastic Hamilton equations
- Phase space area can in principle be calculated numerically
- May present a first quantitative description of cosmological decoherence

For more details see Weenink, Prokopec arXiv:1108.3994