The overshoot problem in inflation after tunneling

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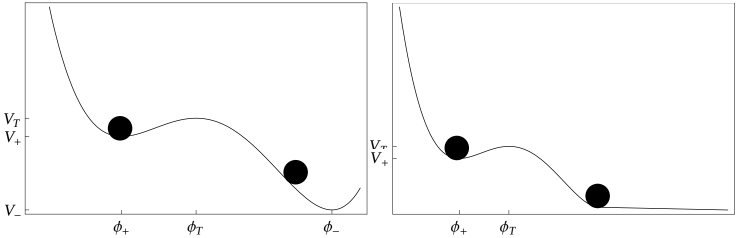
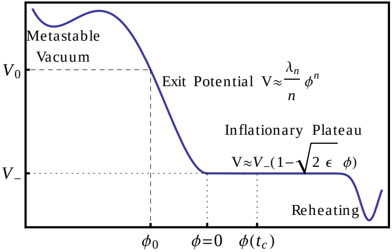
Outline

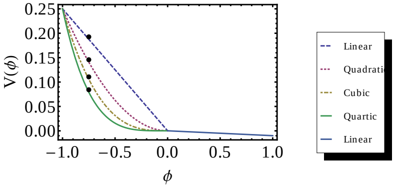
Motivation



- large field models (detectable gravity waves)
- small field models (undetectable gravity waves)
- relative frequency of potentials of either shape
- populating the initial condition
- ignore measure problem
- best observations will be able to detect $r \approx 0.01$
- Lyth bound $\Delta\phi > \sqrt{r \Delta N} M_P$

Setup



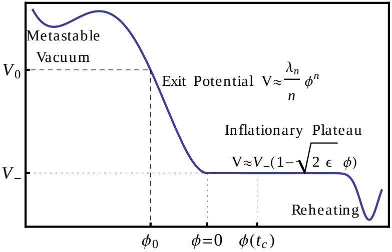


- tunnel from metastable vacuum to ϕ_0 on the "exit potential"
- roll down towards inflationary plateau (linear potential)

$$V = V_-(1 - \sqrt{2\epsilon}\phi)$$

- how far will the field go for ϕ^n monomial exit potentials? (not far!)
- rather: how far until it reaches slow roll?

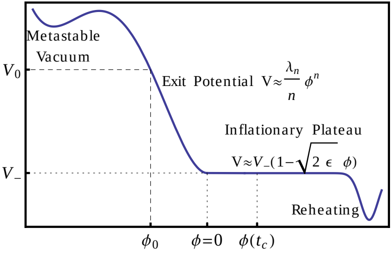
Equation of Motion



$$\ddot{\phi} + \frac{3}{t}\dot{\phi} + \partial_\phi V = 0 \qquad (1)$$

- open universe after tunneling
- assume curvature domination $H = \frac{1}{t}$
- in general, difficult to solve

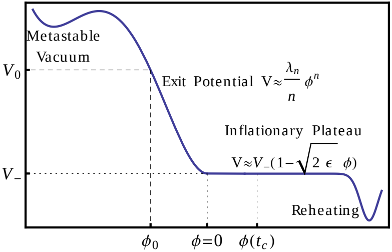
Linear exit potential $V = V_-(1 - \lambda_1\phi)$



$$\ddot{\phi} + \frac{3}{t}\dot{\phi} - \lambda_1 V_- = 0 \qquad (2)$$

- solved by $\phi = \phi_0 + \frac{\lambda_1 V_-}{8} t^2$
- terminal velocity $\dot{\phi}_f = \sqrt{-\frac{1}{2}\lambda_1 V_- \phi_0}$.
- on the plateau, slow roll starts at $\phi(t_c) = \frac{3}{2\sqrt{2}} \frac{\epsilon}{-} \phi_0$
- curvature domination for $|\phi_0| < 1$

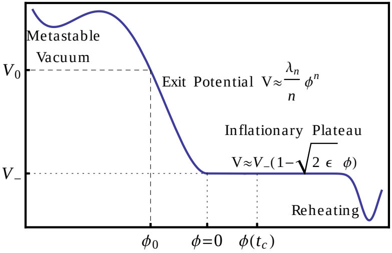
Quadratic exit potential $V = V_- + \frac{1}{2}m^2\phi^2$



$$\ddot{\phi} + \frac{3}{t}\dot{\phi} + m^2\phi = 0 \qquad (3)$$

- solved by $\phi = 2\phi_0 \frac{J_1(mt)}{mt}$ (Bessel function)
- terminal velocity $\dot{\phi}_f = \phi_0 \frac{J_0(mt_f) - J_1(mt_f)}{t_f} \approx -0.21 m\phi_0$, mt_f first zero of J_1
- on the plateau, slow roll starts at $\phi(t_c) = \frac{3}{2\sqrt{2}}\frac{\epsilon}{+}\phi_0 J_0(mt_f) \approx \frac{3}{2\sqrt{2}}\frac{\epsilon}{-}0.403\phi_0$
- curvature domination for $|\phi_0| < 1$ and $V_- \ll V_0$

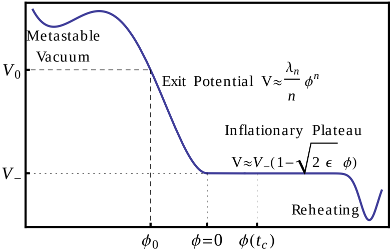
Cubic exit potential $V = V_- - \frac{1}{3}\lambda_3\phi^3$



$$\ddot{\phi} + \frac{3}{t}\dot{\phi} - \lambda_3\phi^2 = 0 \qquad (4)$$

- can't find solution
- BUT: can find solution for larger friction $\frac{10}{3t}$ and $\frac{5}{3t}$

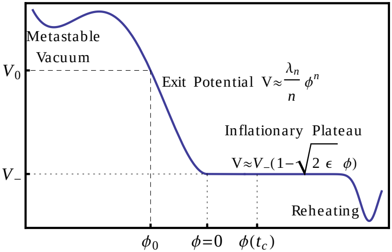
Cubic exit potential $V = V_- - \frac{1}{3}\lambda_3\phi^3$, larger friction



$$\ddot{\phi} + \frac{10}{3t}\dot{\phi} - \lambda_3\phi^2 = 0 \qquad (5)$$

- solved by $\phi = -\frac{2}{3\lambda_3 t^2}\left[z(t)^2\wp\left(\imath z(t), 0, 1\right) + 1\right]$ with $z(t) = (-42\lambda_3\phi_0 t^2)^{1/6}$ and elliptic Weierstrass function $\wp$
- terminal velocity $\dot{\phi}(t_f) \approx -0.03\phi_0\sqrt{-\lambda_3\phi_0}$
- on the plateau, slow roll starts at $\phi(t_c) = \frac{3}{4\sqrt{2}}\sqrt{\epsilon} - 0.1\phi_0$.

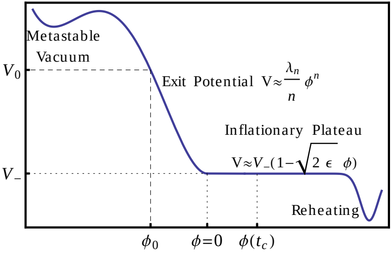
Cubic exit potential $V = V_- - \frac{1}{3}\lambda_3\phi^3$, smaller friction



$$\ddot{\phi} + \frac{5}{3t}\dot{\phi} - \lambda_3\phi^2 = 0 \qquad (6)$$

- solved by $\phi = (8/3\lambda_3 t^2)^{1/3}\mathfrak{p}\left((3t^2\lambda_3/8)^{1/3} + \zeta, 0, \xi\right)$ with $\zeta = 2^{1/3}\frac{\Gamma(\frac{1}{3})\Gamma(\frac{7}{6})}{\sqrt{\pi}|\phi_0|^{1/3}}$, $\xi = -\phi_0^2$ and elliptic Weierstrass function $\mathfrak{p}$
- terminal velocity $\dot{\phi}(t_f) \approx -0.174\phi_0\sqrt{-\lambda_3\phi_0}$
- on the plateau, slow roll starts at $\phi(t_c) = \frac{3}{4\sqrt{2}}\sqrt{\epsilon} - \frac{1}{3}\phi_0$.
- curvature dominates for $|phi_0| < M_p$

Cubic exit potential $V = V_- - \frac{1}{3}\lambda_3\phi^3$, correct friction

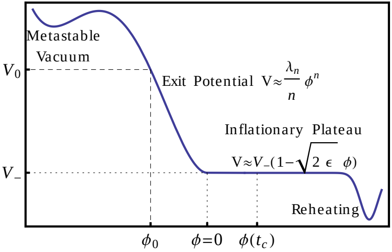


$$\ddot{\phi} + \frac{3}{t}\dot{\phi} - \lambda_3\phi^2 = 0 \qquad (7)$$

- can't find solution
- for larger friction $\frac{10}{3t}$: $\phi(t_c) = \frac{3}{4\sqrt{2}}\sqrt{\epsilon} - 0.1\phi_0$
- for smaller friction $\frac{5}{3t}$: $\phi(t_c) = \frac{3}{4\sqrt{2}}\sqrt{\epsilon} - \frac{1}{3}\phi_0$
- true position when slow roll starts:

$$\frac{3}{4\sqrt{2}}\sqrt{\epsilon} - 0.1\phi_0 < \phi(t_c) < \frac{3}{4\sqrt{2}}\sqrt{\epsilon} - \frac{1}{3}\phi_0$$

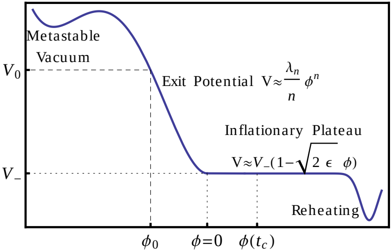
Quartic exit potential $V = V_{-} + \frac{1}{4}\lambda_4\phi^4$



$$\ddot{\phi} + \frac{3}{t}\dot{\phi} + \lambda_4\phi^3 = 0 \qquad (8)$$

- solved by $\phi = \frac{8\phi_0}{8+t^2\lambda_4\phi_0^2}$
- terminal velocity $\dot{\phi}_f = 0.$
- on the plateau, slow roll starts at $\phi(t_c) = \frac{3}{2\sqrt{2}}\frac{\epsilon}{-}\phi_0$
- curvature domination

Higher power exit potential $V = V_- + \frac{1}{n}\lambda_n\phi^n, n > 4$



$$\ddot{\phi} + \frac{3}{t}\dot{\phi} + \lambda_n\phi^n = 0 \quad (9)$$

$$\ddot{\phi} + \frac{\gamma}{t}\dot{\phi} + \phi^n = 0 \quad (10)$$

- don't know how to solve
- BUT: know first integral for equation with smaller friction term $\gamma = \frac{n+2}{n-2}$

$$C = \frac{t^{\gamma-1}}{2}\left(\dot{\phi}^2 t^2 + \dot{\phi}\phi t(\gamma - 1)\right) + \frac{1}{n}\left(\phi t^{\frac{\gamma-1}{2}}\right)^n$$

- at $t = 0$, $\phi, \dot{\phi} < \infty \Rightarrow C = 0$
- at $t = t_f$, $\phi(t_f) = 0 \Rightarrow \dot{\phi}(t_f) = 0$
- curvature domination

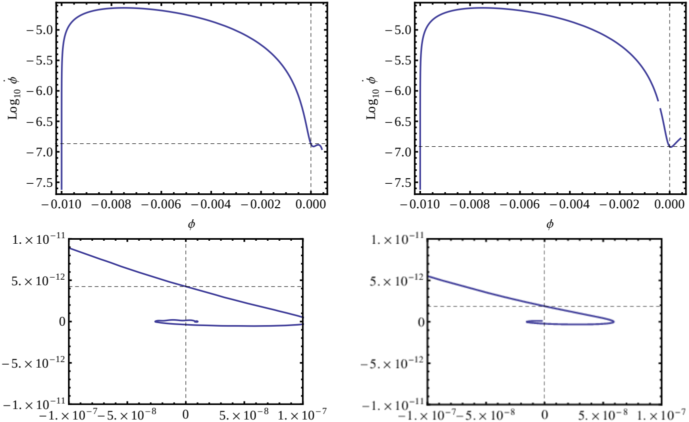
Polynomial exit potential $V = (-1)^m \frac{\lambda_m}{m} \phi^m + (-1)^n \frac{\lambda_n}{n} \phi^n$

$$(-1)^m \lambda_m \phi^{m-1} (1+\delta) \Big|_{\phi=\phi_*} = (-1)^n \lambda_n \phi^{n-1} \Big|_{\phi=\phi_*} \tag{11}$$

$$\frac{t_*}{t_*^{(0)}} = 1 + \frac{\phi_0}{6(\phi_*^{(0)} - \phi_0)} \delta \,, \quad \frac{\dot{\phi}_*}{\dot{\phi}_*^{(0)}} = 1 + \frac{\phi_0 - \frac{4}{3}\phi_*^{(0)}}{2(\phi_0 - \phi_*^{(0)})} \delta \tag{12}$$

- assume $m < n \leq 4$
- effective field theory: $m, n \leq 4$, otherwise suppressed ($|\phi_0| < M_p$)
- glue solutions for ϕ^m, ϕ^n
- small perturbation of the matching location $\Rightarrow$ small perturbation of $\dot{\phi}$ $\Rightarrow$ small change in initial conditions for second part of the potential

Polynomial Examples

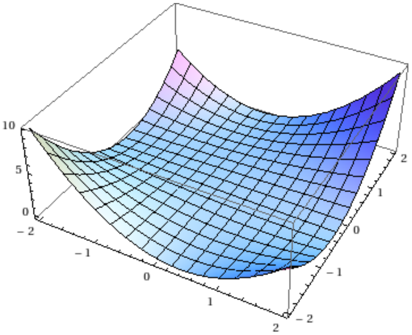


$$V_- - \lambda_1 \phi + \frac{\lambda_4}{4}\phi^4$$

$$V_- + \frac{1}{2}m^2\phi^2 + \frac{\lambda_4}{4}\phi^4$$

overshoot controlled by lowest-order term

Non-zero initial speed

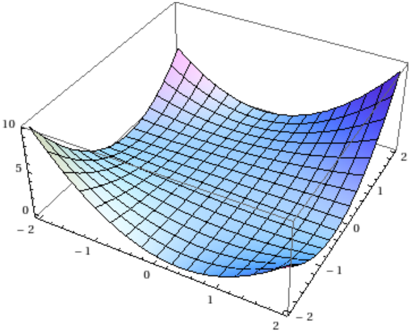


$$\ddot{\phi} = -\frac{3}{t}\dot{\phi} - \partial_\phi V \qquad (13)$$

$$\Rightarrow \dot{\phi} = \dot{\phi}_\epsilon \left(\frac{\epsilon}{t}\right)^3 \qquad (14)$$

- non-zero initial speed after tunneling
- possible in multi-field setups
- immediate slow-down

Non-zero initial speed

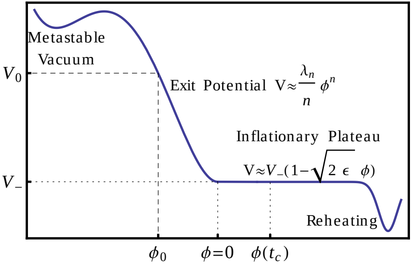


$$\ddot{\phi} = -\frac{3}{t}\dot{\phi} - \partial_\phi V \quad (15)$$

$$\Rightarrow \dot{\phi} = \dot{\phi}_\epsilon \left(\frac{\epsilon}{t}\right)^3 \quad (16)$$

- tunneling not directly into today's vacuum, but somewhere up the inflationary slope
- could have non-zero initial speed
- no overshoot problem either for effective single-field evolution

Conclusions and Outlook



$$
\begin{aligned}
V &= \frac{\lambda_n}{n}\phi^n \\
\Delta\phi &= \frac{3}{4\sqrt{2}}\sqrt{\epsilon} + C|\phi_0| \\
C &= \{1, 0.53, [0.33, 0.1]\}
\end{aligned}
$$

- finite small overshoot for $n = 1 \ldots 3$
- no overshoot for $n \geq 4$
- for polynomials, the lowest order term controls the overshoot