Three-loop anomalous dimensions for squarks in SUSY-QCD

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Introduction

- Supersymmetric extension of QCD $\rightarrow$ gluinos and squarks
- Regularisation scheme
  - *Dimensional reduction*, *dimensional regularisation* breaks SUSY explicitly
  - New unphysical particles: ϵ scalars; mass term in $\mathcal{L}$
  - Minimal renormalization scheme: $\overline{\text{DR}}$ scheme
- Renormalization constants for SUSY-QCD
  - SUSY-QCD β function and anomalous dimensions for quarks and gluinos to three loops [Harlander, Mihaila, Steinhauser 2009]
  - Renormalization constants for squarks to two loops [Kant, Harlander, Mihaila, Steinhauser 2010]
  - New: For squarks to three-loop order
- Anomalous dimensions for masses [Barger, ..., Yamada 1994-2008]
  - running from the GUT scale to the EW scale
  - e.g. constrained MSSM

Top squark sector

$\tilde{t}_L$ and $\tilde{t}_R$ are interaction eigenstates but not mass eigenstates:

$$\begin{pmatrix} m_t^2 + M_Z^2\left(\frac{1}{2} - \frac{2}{3}\sin^2\vartheta_W\right)\cos 2\beta + M_{\tilde{Q}}^2 & m_t\left(A_t - \mu_{\text{SUSY}}\cot\beta\right) \\ m_t\left(A_t - \mu_{\text{SUSY}}\cot\beta\right) & m_t^2 + \frac{2}{3}M_Z^2\sin^2\vartheta_W\cos 2\beta + M_{\tilde{U}}^2 \end{pmatrix}$$

$$\equiv \begin{pmatrix} m_{\tilde{t}_L}^2 & m_t X_t \\ m_t X_t & m_{\tilde{t}_R}^2 \end{pmatrix} = \mathcal{M}_{\tilde{t}}^2$$

Soft SUSY breaking masses $M_{\tilde{Q}}$, $M_{\tilde{U}}$ and soft SUSY breaking tri-linear coupling A_t

- Mass eigenstates $\tilde{t}_{1,2}$ through unitary transformation

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = R_{\tilde{t}}^{\dagger} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} \qquad R_{\tilde{t}} = \begin{pmatrix} \cos\theta_t & -\sin\theta_t \\ \sin\theta_t & \cos\theta_t \end{pmatrix}$$

- Mixing angle θ_t

Renormalization constants for squarks I

- Loop induced transition from $\tilde{t}_1$ to $\tilde{t}_2$ $\rightarrow$ counterterm is needed
- Matrix ansatz for renormalization constants

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}^{(0)} = \mathcal{Z}_{\tilde{t}}^{1/2} \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}$$

$$\mathcal{Z}_{\tilde{t}}^{1/2} = \tilde{Z}_2^{1/2} \begin{pmatrix} \cos\delta\theta_t & \sin\delta\theta_t \\ -\sin\delta\theta_t & \cos\delta\theta_t \end{pmatrix}$$

- Renormalization of the mixing angle $\theta_t^{(0)} \rightarrow \theta_t + \delta\theta_t$
- Renormalization of the masses

$$\begin{pmatrix} m_{\tilde{t}_1}^2 & 0 \\ 0 & m_{\tilde{t}_2}^2 \end{pmatrix}^{(0)} \rightarrow \begin{pmatrix} m_{\tilde{t}_1}^2 Z_{m_{\tilde{t}_1}} & 0 \\ 0 & m_{\tilde{t}_2}^2 Z_{m_{\tilde{t}_2}} \end{pmatrix} \equiv \mathcal{M}$$

Renormalization constants for squarks II

- Loop corrections to self energies

$$\hat{\Sigma} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \qquad \Sigma_{ij} =$$

- Inverse squark propagator in matrix form

$$i\,\mathcal{S}^{-1}(p^2) = p^2 \left(\mathcal{Z}_{\tilde{t}}^{1/2}\right)^{\dagger} \mathcal{Z}_{\tilde{t}}^{1/2} - \left(\mathcal{Z}_{\tilde{t}}^{1/2}\right)^{\dagger} \left(\mathcal{M} - \hat{\Sigma}(p^2)\right) \mathcal{Z}_{\tilde{t}}^{1/2}$$

- Renormalization condition in $\overline{\text{DR}}$ scheme: $\quad i\,S_{ij}^{-1}(p^2)\Big|_{pp} = 0$

$\rightarrow$ e.g. equations to order α_s

$$\left\{ \Sigma_{ii}^{(1)} - m_{\tilde{t}_i}^2 \left(\delta\tilde{Z}_2^{(1)} + \delta Z_{m_{\tilde{t}_i}}^{(1)} \right) + p^2 \delta\tilde{Z}_2^{(1)} \right\} \Bigg|_{pp} = 0\,, \quad i = 1,2$$

$$\left\{ \Sigma_{12}^{(1)} - \delta\theta_t^{(1)} \left(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2 \right) \right\} \Bigg|_{pp} = 0$$

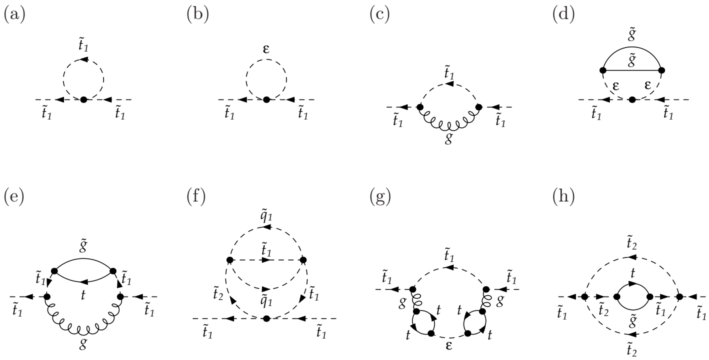
Calculation I

- Squark propagator has mass dimension two $\rightarrow$ renormalization constants depend on the masses (in contrast to quarks and gluinos)
- Generic squark $\tilde{q}$ with mass $m_{\tilde{q}}$
- ϵ scalars with non-vanishing mass m_ϵ, because RGE for the squark masses and m_ϵ are coupled
- Three loop integrals with many different mass scales
  - Asymptotic expansion with EXP $\rightarrow$ Reduction to one scale integrals [Harlander, Seidensticker, Steinhauser 1998 and Seidensticker 1999]
  - MINCER [Larin, Tkachov, Vermaseren 1991] and MATAD package [Steinhauser 2001]
- Calculation with different mass hierarchies
  - $q^2 \gg m_{\tilde{t}_2}^2 \gg m_{\tilde{q}}^2 \gg m_{\tilde{t}_1}^2 \gg m_{\tilde{g}}^2 \gg m_t^2 \gg m_\epsilon^2$
  - $q^2 \gg m_{\tilde{g}}^2 \gg m_{\tilde{q}}^2 \gg m_{\tilde{t}_2}^2 \gg m_{\tilde{t}_1}^2 \gg m_t^2 \gg m_\epsilon^2$
  - $q^2 \gg m_{\tilde{g}}^2 \gg m_{\tilde{q}}^2 \gg m_{\tilde{t}_2}^2 \gg m_t^2 \gg m_{\tilde{t}_1}^2 \gg m_\epsilon^2$

$\rightarrow$ same results

Calculation II

Sample diagrams contributing to Σ_{11}:



- Σ_{11} to 3L ≈ 7000 diagrams
- Σ_{12} to 3L ≈ 5000 diagrams

Calculation III

- Most terms in $\delta\theta_t$ and $Z_{m_{\tilde{t}_i}}$ are proportional to $m^2_{\tilde{t}_i}$, $m^2_{\tilde{g}}$, m^2_t, ...
- But at 2L and 3L order

$$m^2_{\tilde{t}_1}\,\delta Z^{(2)}_{m_{\tilde{t}_1}} \sim \frac{m^2_{\tilde{g}}\,m^2_t}{\left(m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}\right)} = -\frac{m^2_{\tilde{g}}\,m^2_t}{m^2_{\tilde{t}_2}}\sum_{n=0}^{\infty}\left(\frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}}\right)^n$$

$$m^2_{\tilde{t}_1}\,\delta Z^{(3)}_{m_{\tilde{t}_1}} \sim \frac{m^3_{\tilde{g}}\,m^3_t}{\left(m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}\right)^2} = \frac{m^3_{\tilde{g}}\,m^3_t}{m^4_{\tilde{t}_2}}\sum_{n=0}^{\infty}\left(2\,\frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}}-\frac{m^4_{\tilde{t}_1}}{m^4_{\tilde{t}_2}}\right)^n$$

- It was possible to identify the first terms of the geometrical series $\rightarrow$ reconstruction of full mass-dependence
- Asymptotic expansion to $m^{-10}_{\tilde{t}_2}$

Renormalization:

- m_ϵ in the $\overline{\text{DR}}$ and in the on-shell scheme
- all the other parameters (α_s, m_t, $m_{\tilde{g}}$, ...) in the $\overline{\text{DR}}$ scheme

Anomalous dimensions in the $\overline{\text{DR}}$ scheme

- Anomalous dimensions for squark masses

$$m^2_{\tilde{t}_i,0} = m^2_{\tilde{t}_i}\, Z_{m_{\tilde{t}_i}} \quad \Rightarrow \quad \mu^2 \frac{d}{d\mu^2}\, m^2_{\tilde{t}_i} = -\frac{\mu^2}{Z_{m_{\tilde{t}_i}}}\, m^2_{\tilde{t}_i}\, \frac{d}{d\mu^2}\, Z_{m_{\tilde{t}_i}} \equiv \gamma_{m_{\tilde{t}_i}}\, m^2_{\tilde{t}_i}$$

- Anomalous dimension for mixing angle (with $Z_{\theta_t} = 1 + \frac{\delta\theta_t}{\theta_t}$)

$$\theta_{t,0} = \theta_t\, Z_{\theta_t} \quad \Rightarrow \quad \mu^2 \frac{d}{d\mu^2}\, \theta_t = -\frac{\mu^2}{Z_{\theta_t}}\, \theta_t\, \frac{d}{d\mu^2}\, Z_{\theta_t} \equiv \gamma_{\theta_t}\, \theta_t$$

- μ dependence not only in α_s but also in the masses and mixing angle
- $\gamma_{m_{\tilde{t}_i}}$ and γ_{θ_t} are finite quantities $\rightarrow$ important check for the renormalization constants

$\gamma_{m_{\tilde{t}_i}}$ depend on unphysical ϵ scalar mass:

- m_ϵ^{OS} can be set to zero
- If $m_\epsilon^{\overline{\text{DR}}}$ is set to zero at one scale it is different from zero at another scale $\rightarrow$ $\overline{\text{DR}}'$ scheme

Anomalous dimensions in the $\overline{\text{DR}}$ scheme

$$\gamma_{m_{\tilde{t}_1}} = -\frac{\alpha_s}{\pi}\sum_{n\geq 0}\left(\frac{\alpha_s}{\pi}\right)^n \gamma^{(n)}_{m_{\tilde{t}_1}}$$

$$m^2_{\tilde{t}_1}\gamma^{(0)}_{m_{\tilde{t}_1}} = C_F\left[m^2_{\tilde{g}} + \frac{1}{8}(1-c_{4t})\left(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}\right) + m^2_t - m_{\tilde{g}}\, m_t\, s_{2t}\right]$$

$$\begin{aligned}
m^2_{\tilde{t}_1}\gamma^{(2)}_{m_{\tilde{t}_1}} &= C_F^3\left\{3\, m^2_{\tilde{g}} + \frac{1}{2}\, m^2_t - \frac{3}{2}\, m_{\tilde{g}}\, m_t\, s_{2t} + \frac{1}{16}\,(1-c_{4t})\left(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}\right)\right\}\\
&+ C_A^2\, C_F\left\{\frac{45}{32}\, m^2_\epsilon + \frac{15}{4}\, m^2_{\tilde{g}} + \frac{3}{8}\, m^2_t - \frac{9}{8}\, m_{\tilde{g}}\, m_t\, s_{2t} + \frac{3}{64}\,(1-c_{4t})\left(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}\right)\right\}\\
&+ C_F^2\, C_A\left\{-\frac{9}{16}\, m^2_\epsilon - \frac{21}{8}\, m^2_{\tilde{g}} - \frac{3}{8}\, m^2_t + \frac{9}{8}\, m_{\tilde{g}}\, m_t\, s_{2t} - \frac{3}{64}\,(1-c_{4t})\left(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}\right)\right\}\\
&+ C_F\, T_f^2\left\{n_t^2\left[\frac{3}{8}\, m^2_\epsilon - \frac{3}{2}\, m^2_{\tilde{g}} + \frac{3}{4}\, m^2_{\tilde{t}_1} - m^2_t + \frac{3}{4}\, m_{\tilde{g}}\, m_t\, s_{2t} - \frac{1}{32}\,(13-c_{4t})\left(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}\right)\right]\right.\\
&\quad + n_q^2\left[\frac{3}{8}\, m^2_\epsilon - \frac{3}{2}\, m^2_{\tilde{g}} + \frac{3}{4}\, m^2_{\tilde{q}} - \frac{1}{4}\, m^2_t + \frac{3}{4}\, m_{\tilde{g}}\, m_t\, s_{2t} - \frac{1}{32}\,(1-c_{4t})\left(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}\right)\right]\\
&\quad \left. + n_q\, n_t\left[\frac{3}{4}\, m^2_\epsilon - 3\, m^2_{\tilde{g}} + \frac{3}{4}\, m^2_{\tilde{q}} + \frac{3}{4}\, m^2_{\tilde{t}_1} - \frac{5}{4}\, m^2_t + \frac{3}{2}\, m_{\tilde{g}}\, m_t\, s_{2t} + \cdots\right]\right\} + \cdots
\end{aligned}$$

Anomalous dimensions in the $\overline{\text{DR}}'$ scheme

$\overline{\text{DR}}'$ scheme: [Jack, Jones, Martin, Vaughn, Yamada 1994]

same as $\overline{\text{DR}}$ scheme with finite shift in squark masses

$$m_{\tilde{f}}^2 \to m_{\tilde{f}}^2 - \frac{\alpha_s}{\pi}\,\frac{1}{2}\,C_F\,m_\epsilon^2 + \left(\frac{\alpha_s}{\pi}\right)^2 C_F\,m_\epsilon^2 \left(\frac{1}{4}\,T_f\,\left(n_q + n_t\right) + \frac{1}{4}\,C_F - \frac{3}{8}\,C_A\right)$$

[Martin 2002]

Finite term is chosen such that the ϵ scalar mass decouples from the system of differential equations.

$\Rightarrow \quad \gamma_{m_{\tilde{t}_i}}^{\overline{\text{DR}}'}$ are independent of m_ϵ

Anomalous dimensions for soft SUSY breaking parameters

Results in the literature for SUSY anomalous dimensions:

- 1L and 2L order [Barger et al 1994, ..., Martin et al 2008]
- 3L order [Ferreira, Jack, Jones, Kazakov, Kord, North, Velizhanin 1996-2005]
- Full 3L β-functions and anomalous dimensions for MSSM [Jack, Jones, Kord 2005]

Based on so-called NSVZ scheme [Novikov, Shifman, Vainshtein, Zakharov 1983]
Relations between β-functions of gauge and Yukawa couplings and anomalous dimensions of soft breaking parameters

Here: Diagrammatic approach full agreement

Anomalous dimensions for soft SUSY breaking masses $M_{\tilde{Q}}$, $M_{\tilde{U}}$ and soft SUSY breaking tri-linear coupling A_t

$$\begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix} = R^\dagger_{\tilde{t}} \, \mathcal{M}^2_{\tilde{t}} \, R_{\tilde{t}}$$

$$\gamma_{m_{\tilde{t}_1}}, \gamma_{m_{\tilde{t}_2}}, \gamma_{\theta_t} \quad \Leftrightarrow \quad \gamma_{M_{\tilde{Q}}}, \gamma_{M_{\tilde{U}}}, \gamma_{A_t}$$

Numerical example for $\overline{\text{DR}}'$ scheme

Running from $\mu = \mu_G = 10^{16}$ GeV to $\mu = M_Z$

- $m_t(M_Z) = 170$ GeV and $\alpha_s(M_z) = 0.118$
- $\overline{\text{DR}}'$ parameters at the scale $\mu = \mu_G = 10^{16}$ GeV:

$$m_{\tilde{t}_1} = 400 \text{ GeV}\,, \quad m_{\tilde{t}_2} = m_{\tilde{g}} = m_{\tilde{q}} = 600 \text{ GeV}\,, \quad \theta_t = 0.1$$

- $\overline{\text{DR}}'$ parameters of squark sector at the scale $\mu = M_Z$:

	1 loop	2 loops	3 loops	1 loop	2 loops	3 loops
$m_{\tilde{t}_1}$ (GeV)	1425	1416	1378	1456	1419	1378
$m_{\tilde{t}_2}$ (GeV)	1677	1670	1632	1704	1672	1632
θ_t	0.658	0.659	0.656	0.659	0.659	0.656
$m_{\tilde{q}}$ (GeV)	1580	1573	1535	1609	1575	1535

left table:
for α_s, m_t and $m_{\tilde{g}}$
3L running was used

right table:
for all parameters 1L, 2L and
3L running was used

- relatively large 3L corrections

[Jack, Jones, Kord 2005]

Conclusion

Renormalization constants for top-squarks to three-loop order

- Asymptotic expansion $\rightarrow$ reconstruction of full mass dependence

Anomalous dimensions

- $\overline{\text{DR}}$ and $\overline{\text{DR}}'$ scheme
- diagrammatic approach
- full agreement with [Jack, Jones, Kord 2005]
- relatively large 3L corrections

All renormalization constants and anomalous dimensions in `Mathematica` format can be downloaded from

`www-ttp.particle.uni-karlsruhe.de/Progdata/ttp11/ttp11-16/`