Dilaton gravity at the brane with general matter-dilaton coupling

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Outline

beyond General Relativity

- ongoing search for a unified description of
  - gravity
  - gauge interactions of the Standard Model
  - ↪ *string theories* as the most promising proposal
- *low-energy effective action* in string theories
  - *dilaton* (ϕ): a scalar field accompanying gravity
  - at the leading order (when restricted to gravity and the dilaton)
  - ↪ Einstein gravity coupled to the dilaton
- additional spatial dimensions
  - required by the string theories' formulation
  - have to be compactified or warped
  - ↪ dilaton gravity in a 5d brane scenario

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scalar-tensor theories of gravity & conformal frames

- *dilaton gravity*: a scalar-tensor theory of gravity
  - $\hookrightarrow$ can be formulated in various conformally-related frames
  - ⥀ gravitational Lagrangians differ e.g. in the coefficient of the Ricci scalar
    - $\hookrightarrow$ (generically) scalar field dependent coefficients
    - ⥀ Einstein frame: $\mathcal{L} = \frac{1}{2\kappa}\,\mathcal{R} + \cdots$ (coefficient: a constant)
    - ⥀ Jordan frame: e.g. $\mathcal{L} = \frac{1}{16\pi}\,\phi\,\mathcal{R} + \cdots$ (coefficient: a polynomial function of the scalar field)
    - ⥀ string frame: e.g. $\mathcal{L} = e^{-\phi}\,\frac{\alpha_1}{2}\mathcal{R} + \cdots$ (coefficient: an exponential function of the dilaton)
  - ⥀ related ($g_{\mu\nu}$ & $\widetilde{g}_{\mu\nu}$) by a conformal (Weyl) transformation: $g_{\mu\nu} = \Omega(x)^2\,\widetilde{g}_{\mu\nu}$

non-minimal matter-dilaton coupling

- if a matter term $\mathcal{L}_m$ is included into the Lagrangian in one frame
  - conformal transformation to another frame will change its coefficient
  - $\hookrightarrow$ if constant in one frame, it will become dilaton dependent in the other
- which conformal frame is the *natural physical* frame?
  - no clear consensus
  - $\hookrightarrow$ in which frame the matter-dilaton coupling should be minimal?
- thus: a general non-minimal coupling $f(\phi)\,\mathcal{L}_m$ of the *dilaton* to the brane *matter Lagrangian*
  - choice: working in the Einstein frame

Outline

dilaton gravity at the brane with general matter-dilaton coupling

- 5d Lagrangian density:

$$\mathcal{L} = \frac{\alpha_1}{2}\left[\mathcal{R} - \frac{2}{3}\nabla^\sigma \partial_\sigma \phi - \frac{1}{3}(\partial\phi)^2\right] - V(\phi) + f(\phi)\,\mathcal{L}_B\,\delta_B$$

  - $f(\phi)\mathcal{L}_B\,\delta_B$: brane localized term

    $\hookrightarrow$ position of the co-dimension 1 brane: Dirac delta type distribution δ_B

  - 'cosmological constant'-type term (λ) on the brane:

    $f(\phi)\mathcal{L}_B = f(\phi)\mathcal{L}_m + \lambda(\phi)$ ($\mathcal{L}_m$: matter content of the universe)

- induced (projected) brane metric: $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ (covariant approach)
  - n^μ: vector field orthonormal to the brane at its position
  - $g_{\mu\nu}$: $\mathcal{R}_{\mu\nu}{}^{\rho\sigma}$ & ∇_μ vs $h_{\mu\nu}$: $R_{\mu\nu}{}^{\rho\sigma}$ & D_μ

- assume a $\mathbb{Z}_2$ symmetry for the bulk (with its fixed point at the brane position)
  - usually imposed 'automatically'
  - crucial for the existence of the effective brane equations

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at the brane: effective Einstein-like equation

- consequently, the *effective* Einstein-like equation at the brane reads

$$R_{\mu\nu} - \tfrac{1}{2}\, h_{\mu\nu} R = 8\pi G\, \tau_{\mu\nu} - h_{\mu\nu}\Lambda(\phi) + \frac{f^2(\phi)}{4\alpha_1^2}\pi_{\mu\nu} - E_{\mu\nu}$$
$$+ \tfrac{2}{9}\,(\partial_\mu\phi)(\partial_\nu\phi) - \tfrac{5}{36}\, h_{\mu\nu}\,(\partial\phi)^2$$

  - $G = \frac{-1}{48\pi\alpha_1^2} f(\phi)\lambda(\phi)$ (effective brane Newton's constant)
  - $\tau_{\mu\nu} = h_{\mu\nu}\,\mathcal{L}_m - 2\,\frac{\delta\mathcal{L}_m}{\delta h^{\mu\nu}}$, $\tau_\phi = \frac{f'(\phi)}{f(\phi)}\,\mathcal{L}_m + \frac{\delta\mathcal{L}_m}{\delta\phi}$ (brane localized sources)
  - $\Lambda = \frac{1}{2\alpha_1} V - \frac{1}{4\alpha_1^2}\left[\frac{3f^2}{4}\tau_\phi^2 - \frac{1}{3}\lambda^2 + \frac{3}{4}\lambda'^2 + \frac{3f}{2}\lambda'\tau_\phi\right]$ (eff. brane cosmol. const.)
  - $\pi_{\mu\nu} \equiv -\tau_{\mu\rho}\tau^\rho_\nu + \frac{1}{3}\,\tau\,\tau_{\mu\nu} + \frac{1}{2}\,h_{\mu\nu}\tau^\sigma_\rho\tau^\rho_\sigma - \frac{1}{6}\,h_{\mu\nu}\,\tau^2$
  - bulk's influence *on the brane gravity*: $E_{\mu\nu} = n^\alpha h^\beta_\mu n^\gamma h^\delta_\nu\, \mathcal{C}_{\alpha\beta\gamma\delta}$
    (bulk Weyl tensor projected on the brane)

- consistency condition (on the brane sources): $D_\lambda\big(f(\phi)\tau^\lambda_\mu\big) = f(\phi)\,\tau_\phi(\partial_\mu\phi)$

  (brane: 'generalized' covariant conservation of the energy-momentum tensor)

Outline

on the brane: spatial derivative of the energy density

- effective Einstein-like equation at the brane:

$$R_{\mu\nu} - \frac{1}{2} h_{\mu\nu} R = 8\pi G \tau_{\mu\nu} - h_{\mu\nu}\Lambda(\phi) + \frac{f^2(\phi)}{4\alpha_1^2}\pi_{\mu\nu} - E_{\mu\nu} + \frac{2}{9}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{5}{36} h_{\mu\nu}(\partial\phi)^2$$

- assumptions:
  - bulk: anti de Sitter type spacetime: $AdS_5 \rightarrow E_{\mu\nu} = 0$
  - brane: perfect fluid $\rightarrow \tau_{\mu\nu} = \rho_m t_\mu t_\nu + p_m \gamma_{\mu\nu}$ (ρ_m: (dark) matter & radiation)
- calculus ingredients
  - 4d Bianchi identity: $D^\nu\left(R_{\mu\nu} - \frac{1}{2}h_{\mu\nu}R\right) = 0$
  - consistency condition: $D_\lambda\left(f(\phi)\tau_\mu^\lambda\right) = f(\phi)\,\tau_\phi(\partial_\mu\phi)$
- consequently, the spatial derivative of the energy density reads

$$\rho_{m,i} = -\left(\frac{f'}{f}\rho_m - \frac{\lambda'}{f}\right)\phi_{,i} + \frac{\alpha_1^2}{3f^2(\rho_m+p_m)}\left[D^\nu\partial_i\phi - \dot{\phi}^{-1}\phi_{,i}\,D^\nu\partial_t\phi\right](\partial_\nu\phi)$$

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on the brane: late universe

- $\phi \approx \mathrm{const}$ (induces variation of fundamental constants)
  - terms $\mathcal{O}((\partial\phi))$ still treated as non-negligible
  - terms $\mathcal{O}((\partial\phi)D\partial\phi)$ can be dropped, as $\ddot{\phi} \ll \dot{\phi}^2$ expected

(if $\ddot{\phi} \not\ll \dot{\phi}^2$: currently observed $\phi \approx \mathrm{const}$ would be another coincidence problem)

- also: typically $\phi_{,i} \lesssim c_1\,\dot{\phi}$ where $c_1 \sim \mathcal{O}(1)$
  - *spatial variation* of the fundamental constants expected to be smaller than their *time variation*

(any initial inhomogeneities of the dilaton washed out by inflation)

- to start with: $\lambda \neq \lambda(\phi)$

('cosmological constant'-type term in the energy-momentum tensor on the brane)

late universe: spatial derivative of the energy density

- hence for the late universe we obtain

$$\rho_{m,i} \simeq -\frac{f'}{f}\rho_m\,\phi_{,i}$$

spatial inhomogeneities in the matter energy density

are highly constrained

for the ‘popular’ assumptions of *AdS_5 bulk* and *perfect fluid on the brane*

⥀ inhomogeneous perfect fluid ($\rho_{m,i} \neq 0$) on the brane is allowed
only if the *matter on the brane non-minimally coupled to the dilaton* ($f' \neq 0$)

example: observationally allowed values

- experimental limits: $\dot{G}/G < 10^{-12}\ \mathrm{yr}^{-1}$

  (observations: solar system, pulsar timing, CMB, BBN)

  ↪ for $\lambda' = 0 \ \rightarrow \ \frac{f'}{f}\dot{\phi} < 10^{-12}\ \mathrm{yr}^{-1}$

  ⟲ with $\phi_{,i} \lesssim c_1\ \dot{\phi}$

  ↪ observationally allowed values: $\frac{f'}{f}\phi_{,i} \lesssim c_1 \cdot 10^{-12}\,\mathrm{ly}^{-1}$

- thus spatial inhomogeneities in the energy density:

$$\rho_{m,i} \lesssim -c_1\ \rho_m \cdot 10^{-12}\,\mathrm{ly}^{-1}$$

  ↪ to be compared with the observational data!. . .

however: perhaps not so restrictive for $\lambda = \lambda(\phi)$?

on the brane: late universe

- again: the spatial derivative of the energy density reads

$$\rho_{m,i} = -\left(\frac{f'}{f}\rho_m - \frac{\lambda'}{f}\right)\phi_{,i} + \frac{\alpha_1^2}{3f^2(\rho_m+p_m)}\left[D^\nu\partial_i\phi - \dot{\phi}^{-1}\phi_{,i}\,D^\nu\partial_t\phi\right](\partial_\nu\phi)$$

- ingredients: late universe approximations
  - $\phi \approx \mathrm{const}$
    - terms $\mathcal{O}\left((\partial\phi)\right)$ still treated as non-negligible
    - terms $\mathcal{O}\left((\partial\phi)D\partial\phi\right)$ can be dropped
  - $G \approx \mathrm{const}$
    - experimental limits: $\dot{G}/G < 10^{-12}\ \mathrm{yr}^{-1}$
    - $G_{,i} \lesssim c_1\dot{G}$ as $G = G(\phi)$ and $\phi_{,i} \lesssim c_1\,\dot{\phi}$
  - $\hookrightarrow$ $\lambda'(\phi) \approx -\frac{f'(\phi)}{f(\phi)}\lambda(\phi)$

late universe: spatial derivative of the energy density

- hence for the late universe we obtain

$$\rho_{m,i} \simeq -\frac{f'}{f}\left(\rho_m + \frac{\lambda}{f}\right)\phi_{,i}$$

↪ similar conclusions as for the $\lambda \neq \lambda(\phi)$ case!

- spatial inhomogeneities in the matter energy density are highly constrained for the ‘popular’ assumptions of *AdS_5 bulk* and *perfect fluid on the brane*

↪ inhomogeneous perfect fluid ($\rho_{m,i} \neq 0$) on the brane is allowed only if the *matter on the brane non-minimally coupled to the dilaton* ($f' \neq 0$)

- inhomogeneities in the matter energy density suppressed by small $\phi_{,i}$:
  - $\dot{\phi}_0 \lesssim 2.5\, H_0 \approx 1.8 \cdot 10^{-10}\ \mathrm{yr}^{-1}$
    - (model-independent) bound set by current observational data
    - analysis: deceleration parameter $q_0 < 0$ for $[\rho_m^2] \ll [\rho_m]$ (in modified Friedmann eq.) with $\Omega_{tot} = \Omega_m + \Omega_{\overline{\Lambda}} + \Omega_\phi = 1$, $\Omega_{m0} > 0.2$, $H_0 \approx 70\,\frac{\mathrm{km}}{\mathrm{s \cdot Mpc}}$
  - $\phi_{,i} \lesssim c_1\, \dot{\phi}$, where $c_1 \sim \mathcal{O}(1)$

↪ $\phi_{0,i} \lesssim 1.8\, c_1 \cdot 10^{-10}\ \mathrm{ly}^{-1}$

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  $\hookrightarrow$ $\phi_{0,i} \lesssim 1.8\, c_1 \cdot 10^{-10}\ \mathrm{ly}^{-1}$

example: maximal observationally allowed values

- ingredients (late universe):
  - $\phi_{0,i} \approx 1.8\, c_1 \cdot 10^{-10}\ \mathrm{ly}^{-1}$ (max.)
  - observations: $\sum_i \Omega_i = 1$ up to 2%

    dilaton gravity at the brane - *modified* Friedmann equation:

    $\sum_i \Omega_i = 1 + \Omega_m \frac{f\langle \rho_m \rangle}{2\lambda}$ (AdS$_5$, perfect fluid; $\Omega_m = \langle \rho_m \rangle / \rho_c$)

    thus for $\Omega_m \frac{f\langle \rho_m \rangle}{2\lambda} \approx 2\%$ (max.) $\longrightarrow$ $f\langle \rho_m \rangle / \lambda \approx 0.14$ (for $\Omega_m = 0.28$)
- thus spatial inhomogeneities in the energy density:

$$\rho_{m,i} \simeq -\frac{f'}{f} \left(0.14\, \rho_m + \langle \rho_m \rangle\right) 1.3\, c_1 \cdot 10^{-9}\, \mathrm{ly}^{-1}$$

  - a bit less restrictive than for $\lambda \neq \lambda(\phi)$ ...

  $\hookrightarrow$ but will it be possible to adjust f'/f to fit our universe?...

Outline

conclusions & outlook

- dilaton gravity addressed in a 5d brane scenario
- brane: non-minimal dilaton–matter coupling $f(\phi)$
- assumptions (popular in the literature):
  - bulk: anti de Sitter type spacetime
  - brane: perfect fluid (matter content of the universe)

⇝ energy density inhomogeneities *constrained*

- non-minimal dilaton–matter coupling *essential*

↪ $\lambda \neq \lambda(\phi)$: an upper bound on $\rho_{m,i}$,

to be compared straight away with the observational data!

↪ $\lambda = \lambda(\phi)$: can f'/f be appropriately *adjusted*,

so that our universe's structures can be described?

- or: no pure AdS_5 bulk if on the brane minimal matter-dilaton coupling?