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INFLATION AND NONMINIMAL SCALAR-CURVATURE COUPLING IN GRAVITY AND SUPERGRAVITY

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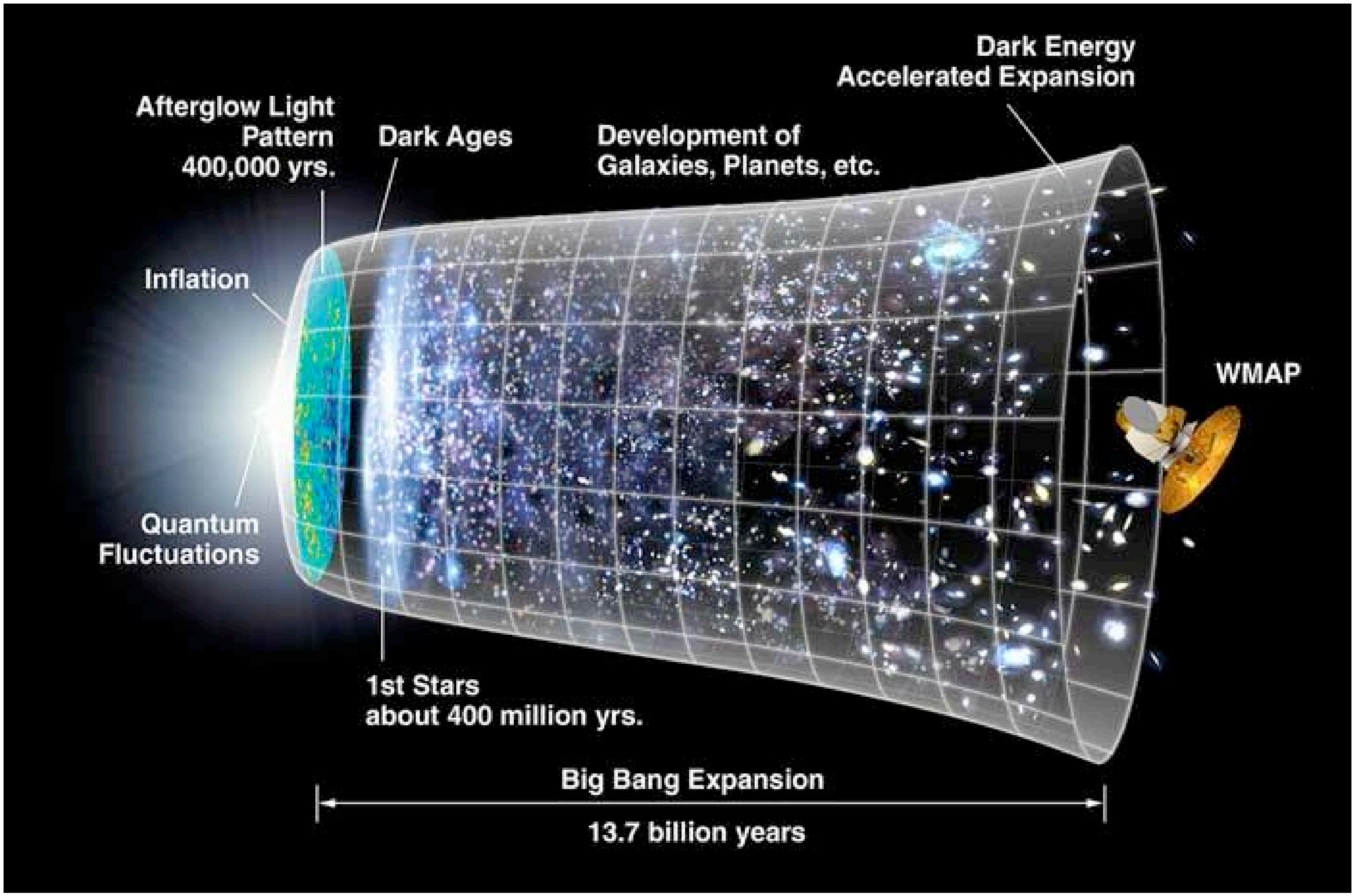
History of our Universe



Inflation in Early Universe

- Cosmological inflation (a phase of ‘rapid’ accelerated expansion) predicts homogeneity of our Universe at large scales, its spatial flatness, large size and entropy, and the almost scale-invariant spectrum of cosmological perturbations (in good agreement with the WMAP measurements of the CMB radiation spectrum)

- Inflation is a paradigm, not a theory! Known theoretical mechanisms of inflation use a slow-roll scalar field (called inflaton) with proper scalar potential

- The scale of inflation is well beyond the electro-weak scale, ie. well beyond the SM ! Inflationary stage in the early Universe is the most powerful HEP accelerator in the Nature ($> 10^{10}\ TeV$). Inflation is a great window to HEP!

- The nature of the inflaton and the origin of its scalar potential are the big mysteries. Knowing the origin of inflaton implies knowing its interactions which lead to definite physical predictions about inflation.

Higgs (with nonmiminal coupling to gravity) as the inflaton

- was proposed by Bezrukov and Shaposhnikov (2008), assuming no new physics beyond the Standard Model up to the Planck scale.

- The nonminimal coupling is required by quantum renormalization in curved spacetime.

- We assume that there is the new physics beyond the Standard Model, and it is given by supersymmetry. Then it is natural to search for the most economical mechanisms of inflation (in particular, with Higgs as the inflaton) in the context of supergravity.

Review of Higgs inflation (I)

Consider the 4D Lagrangian

$$\mathcal{L}_{\mathrm{J}} = \sqrt{-g_{\mathrm{J}}}\left\{-\frac{1}{2}(1+\xi\phi_{\mathrm{J}}^2)R_{\mathrm{J}} + \frac{1}{2}g_{\mathrm{J}}^{\mu\nu}\partial_\mu\phi_{\mathrm{J}}\partial_\nu\phi_{\mathrm{J}} - V(\phi_{\mathrm{J}})\right\} \tag{1}$$

where we have introduced the real scalar field $\phi_{\mathrm{J}}(x)$, nonminimally coupled to gravity (with the coupling constant ξ) in Jordan frame, with the Higgs-like scalar potential

$$V(\phi_{\mathrm{J}}) = \frac{\lambda}{4}(\phi_{\mathrm{J}}^2 - v^2)^2 \tag{2}$$

We use the units $\hbar = c = M_{\mathrm{Pl}} = 1$, where M_{Pl} is the reduced Planck mass, with the spacetime signature $(+,-,-,-)$.

The action (1) can be rewritten to Einstein frame by redefining the metric via a Weyl transformation,

$$g^{\mu\nu} = \frac{g_{\mathrm{J}}^{\mu\nu}}{(1+\xi\phi_{\mathrm{J}}^2)} \tag{3}$$

Review of Higgs inflation (II)

Then one gets the standard Einstein-Hilbert term $(-\frac{1}{2}R)$ for gravity. However, it also leads to a nonminimal kinetic term of the scalar field $\phi_{\rm J}$. To get the canonical kinetic term, a scalar field redefinition is needed, $\phi_{\rm J} \to \varphi(\phi_{\rm J})$, subject to the condition

$$\frac{d\varphi}{d\phi_{\rm J}} = \frac{\sqrt{1+\xi(1+6\xi)\phi_{\rm J}^2}}{1+\xi\phi_{\rm J}^2} \tag{4}$$

As a result, the nonminimal theory (1) is classically equivalent to the standard (canonical) theory of the scalar field $\varphi(x)$ minimally coupled to gravity,

$$\mathcal{L}_{\rm E} = \sqrt{-g}\left\{-\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right\} \tag{5}$$

with the scalar potential

$$V(\varphi) = \frac{V(\phi_{\rm J}(\varphi))}{[1+\xi\phi_{\rm J}^2(\varphi)]^2} \tag{6}$$

Review of Higgs inflation (III)

Given a large positive $\xi \gg 1$, in the small field limit one finds from eq. (4) that $\phi_{\text{J}} \approx \varphi$, whereas in the large φ limit one gets

$$\varphi \approx \sqrt{\frac{3}{2}} \log(1 + \xi \phi_{\text{J}}^2) \tag{7}$$

Equation (6) then yields a scalar potential:
(i) in the *very small* field limit, $\varphi < \sqrt{\frac{2}{3}} \xi^{-1}$, as

$$V_{\text{vs}}(\varphi) \approx \frac{\lambda}{4} \varphi^4 \tag{8}$$

(ii) in the *small* field limit, $\sqrt{\frac{2}{3}} \xi^{-1} < \varphi \ll \sqrt{\frac{3}{2}}$, as

$$V_{\text{s}}(\varphi) \approx \frac{\lambda}{6\xi^2} \varphi^2, \tag{9}$$

Review of Higgs inflation (IV)

(iii) and in the *large* field limit, $\varphi \gg \sqrt{\frac{2}{3}}\xi^{-1}$, as

$$V(\varphi) \approx \frac{\lambda}{4\xi^2}\left(1 - \exp\left[-\sqrt{\frac{2}{3}}\varphi\right]\right)^2 \tag{10}$$

We have assumed here that $\xi \gg 1$ and $v\xi \ll 1$.

Identifying inflaton with Higgs particle requires the parameter v to be the order of weak scale, and the coupling λ be the Higgs boson selfcoupling at the inflationary scale. The Higgs-like scalar potential is perfectly suitable to support a slow-roll inflation, while its consistency with the COBE normalization condition for the observed CMB amplitude of density perturbations (eg., at the e-foldings number $N_e = 50 \div 60$) gives rise to $\xi/\sqrt{\lambda} \approx 10^4 \div 10^5$. The scalar potential (9) corresponds to the post-inflationary matter-dominated epoch with the oscillating inflaton field φ of the frequency

$$\omega = \sqrt{\frac{\lambda}{3}}\xi^{-1} \tag{11}$$

Inflation in Starobinsky model

Viable inflationary models can be also easily constructed in $f(R)$-gravity theories,

$$S = \int d^4x\sqrt{-g}\, f(R) \tag{12}$$

whose function $f(R)$ begins with the Einstein-Hilbert term, while the rest takes the form $R^2C(R)$ for $R \to \infty$, with a slowly varying function $C(R)$. The simplest (Starobinsky) model is given by $C(R) = const. \neq 0$ with

$$f(R) = -\frac{1}{2}\left(R - \frac{R^2}{6M^2}\right) \tag{13}$$

It is well known as the excellent model of chaotic inflation. M actually coincides with the rest mass of the scalar particle (scalaron/inflaton) appearing in $f(R)$ gravity. The model fits the observed amplitude of scalar perturbations if $M/M_{\rm Pl} \approx 1.5 \cdot 10^{-5}(50/N_e)$, and gives rise to the spectral index $n_s - 1 \approx -2/N_e \approx -0.04(50/N_e)$ and the scalar-to-tensor ratio $r \approx 12/N_e^2 \approx 0.005(50/N_e)^2$, in terms of the e-foldings number $N_e \approx (50 \div 55)$ depending upon details of reheating after inflation. The model (13) remains viable, being in agreement with the WMAP7 observations of $n_s = 0.963 \pm 0.012$ and $r < 0.24$ (with 95% CL).

$f(R)$ gravity and quintessence (I)

$f(R)$ gravity theory is classically equivalent to the scalar-tensor gravity. In order to derive the corresponding scalar potential, one rewrites the theory (12) to the equivalent form

$$S_A = \int d^4x\sqrt{-g}\,[AR - Z(A)] \tag{14}$$

where the 'Lagrange multiplier' A has been introduced. Via eliminating the scalar field A by its equation of motion from the action (14) one gets back the original action (12) provided that the functions f and Z are related via Legendre transformation,

$$f(R) = RA(R) - Z(A(R)) \tag{15}$$

It follows, in particular, that

$$Z'(A) = R \qquad \text{and} \qquad f'(R) = A \tag{16}$$

where the primes denote the derivatives with respect to the given argument.

$f(R)$ gravity and quintessence (II)

A Weyl transformation

$$g_{\mu\nu} \to g_{\mu\nu} \exp\left[-\sqrt{\frac{2}{3}}\varphi\right] \tag{17}$$

with the conformal factor

$$\exp\left[\sqrt{\frac{2}{3}}\varphi\right] = A \tag{18}$$

allows us to bring the action (14) to the Einstein frame with the canonical kinetic terms,

$$S_\varphi = \int d^4x\,\sqrt{-g}\left\{-\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{2}\exp\left[\frac{-4\varphi}{\sqrt{6}}\right] Z(A(\varphi))\right\} \tag{19}$$

in terms of the physical (and canonically normalized) scalar field φ, with the scalar potential

$$V(\varphi) = -\frac{1}{2}\exp\left[\frac{-4\varphi}{\sqrt{6}}\right] Z\left(\exp\left[\sqrt{\frac{2}{3}}\varphi\right]\right) \tag{20}$$

Quintessence in the the Starobinsky case

In the special case

$$f_{\mathsf{S}}(R) = -\frac{1}{2}\left(R - \frac{1}{6M^2}R^2\right) \tag{21}$$

one finds

$$V(\varphi) = \frac{3}{4}M^2\left(1 - \exp\left[-\sqrt{\frac{2}{3}}\varphi\right]\right)^2 \tag{22}$$

This inflaton scalar potential is the same as the one in eq. (10) provided that we identify the couplings as

$$3M^2 = \frac{\lambda}{\xi^2} \tag{23}$$

Therefore, the inflationary dynamics in the Higgs inflation and the Starobinsky inflation are essentially the same. In particular, the inflaton mass is given by

$$M = \frac{1}{\xi}\sqrt{\frac{\lambda}{3}} = \omega \tag{24}$$

Nonmiminal coupling in supergravity (I)

In 4D, N=1 supersymmetry, gravity is to be extended to N=1 supergravity, while a scalar field should be complexified and become the leading complex scalar field component of a chiral (scalar) matter supermultiplet. In a curved superspace of N=1 supergravity, the chiral matter supermultiplet is described by a covariantly chiral superfield Φ obeying the constraint $\overline{\nabla}_{\stackrel{\bullet}{\alpha}}\Phi = 0$ in the notation of Wess and Bagger. The *standard* (generic and minimally coupled) matter-supergravity action reads

$$S_{\rm MSG} = -3\int d^4x d^4\theta E^{-1} \exp\left[-\frac{1}{3}K(\Phi,\overline{\Phi})\right] + \left\{\int d^4x d^2\theta \mathcal{E} W(\Phi) + {\rm H.c.}\right\} \tag{25}$$

in terms of the Kähler potential K and the superpotential W of the chiral supermatter, the full density E and the chiral density $\mathcal{E}$ of the superspace supergravity. It is convenient to introduce the notation

$$\Omega = -3\exp\left[-\frac{1}{3}K\right] \quad \text{or} \quad K = -3\ln\left[-\frac{1}{3}\Omega\right] \tag{26}$$

Nonmiminal coupling in supergravity (II)

The nonminimal matter-supergravity coupling in superspace reads

$$S_{\text{NM}} = \int d^4x d^2\theta \mathcal{E} X(\Phi) \mathcal{R} + \text{H.c.} \tag{27}$$

in terms of the chiral function $X(\Phi)$ and the N=1 chiral scalar supercurvature superfield $\mathcal{R}$ obeying $\overline{\nabla}_{\overset{\bullet}{\alpha}} \mathcal{R} = 0$. In terms of the field components of the superfields the nonminimal action (27) is given by

$$\int d^4x d^2\theta \mathcal{E} X(\Phi) \mathcal{R} + \text{H.c.} = -\frac{1}{6} \int d^4x \sqrt{-g} X(\phi_c) R + \text{H.c.} + \ldots \tag{28}$$

where the dots stand for the fermionic terms, and $\phi_c = \Phi| = \phi + i\chi$ is the leading complex scalar field component of the superfield Φ. Given $X(\Phi) = -\xi\Phi^2$ with the real coupling constant ξ, we find the bosonic contribution

$$S_{\text{NM,bos.}} = \frac{1}{6}\xi \int d^4x \sqrt{-g} \left(\phi^2 - \chi^2\right) R \tag{29}$$

The supersymmetrizable non-minimal coupling reads $\left[\phi_c^2 + (\phi_c^\dagger)^2\right] R$, and not $(\phi_c^\dagger \phi_c) R$.

Nonminimal coupling in supergravity (III)

The manifestly supersymmetric nonminimal action (in Jordan frame) reads

$$S = S_{\mathrm{MSG}} + S_{\mathrm{NM}} \tag{30}$$

In curved superspace of N=1 supergravity the (Siegel's) chiral integration rule

$$\int d^4x d^2\theta \mathcal{E} \mathcal{L}_{\mathrm{ch}} = \int d^4x d^4\theta E^{-1} \frac{\mathcal{L}_{\mathrm{ch}}}{\mathcal{R}} \tag{31}$$

applies to any chiral superfield Lagrangian $\mathcal{L}_{\mathrm{ch}}$ with $\overline{\nabla}_{\stackrel{\bullet}{\alpha}} \mathcal{L}_{\mathrm{ch}} = 0$. It is, therefore, possible to rewrite eq. (30) to the equivalent form

$$S_{\mathrm{NM}} = \int d^4x d^4\theta E^{-1} \left[X(\Phi) + \overline{X}(\overline{\Phi}) \right] \tag{32}$$

We conclude that adding S_{NM} to S_{MSG} is equivalent to the simple change of the Ω-potential as

$$\Omega \to \Omega_{\mathrm{NM}} = \Omega + X(\Phi) + \bar{X}(\overline{\Phi}) \tag{33}$$

Because of eq. (26), it amounts to the change of the Kähler potential as

$$K_{\mathrm{NM}} = -3 \ln \left[e^{-K/3} - \frac{X(\Phi) + \overline{X}(\overline{\Phi})}{3} \right] \tag{34}$$

Nonminimal coupling in supergravity (IV)

The scalar potential in the matter-coupled supergravity (25) is given by

$$V(\phi,\bar{\phi}) = e^G \left[\left(\frac{\partial^2 G}{\partial\phi\partial\bar{\phi}} \right)^{-1} \frac{\partial G}{\partial\phi}\frac{\partial G}{\partial\bar{\phi}} - 3 \right] \tag{35}$$

in terms of the *single* (Kähler-gauge-invariant) function

$$G = K + \ln |W|^2 \tag{36}$$

Hence, in the nonminimal case (30) we have

$$G_{\text{NM}} = K_{\text{NM}} + \ln |W|^2 \tag{37}$$

Contrary to the bosonic case, one gets a nontrivial Kähler potential K_{NM}, ie. a *Non-Linear Sigma-Model* (NLSM) as the kinetic term of $\phi_c = \phi + i\chi$. Since the NLSM target space has a nonvanishing curvature, no field redefinition exist that could bring the kinetic term to the free (canonical) form with its Kähler potential $K_{\text{free}} = \overline{\Phi}\Phi$.

$F(\mathcal{R})$ supergravity

$F(\mathcal{R})$ supergravity is the 4D, N=1 supersymmetric extension of $f(R)$ gravity. It is most nicely formulated in a curved chiral superspace (Gates Jr., SVK, 2009),

$$S = \int d^4x d^2\theta\, \mathcal{E} F(\mathcal{R}) + \text{H.c.} \tag{38}$$

in terms of a holomorphic function $F(\mathcal{R})$ of the covariantly-chiral scalar curvature superfield $\mathcal{R}$, and the chiral superspace density $\mathcal{E}$. The chiral $N = 1$ superfield $\mathcal{R}$ has the scalar curvature R as the field coefficient at its θ^2-term. The chiral superspace density $\mathcal{E}$ (in a WZ gauge) reads

$$\mathcal{E} = e\left(1 - 2i\theta\sigma_a\bar{\psi}^a + \theta^2 B\right) \tag{39}$$

where $e = \sqrt{-g}$, ψ^a is gravitino, and $B = S - iP$ is the complex scalar auxiliary field (it does not propagate in the theory (38) despite of the apparent presence of the higher derivatives). The $F(\mathcal{R})$ supergravity is classically equivalent to the standard N=1 Poincaré supergravity minimally coupled to the chiral scalar superfield, via the supersymmetric Legendre-Weyl-Kähler transform (SVK, 2010).

$F(\mathcal{R})$ supergravity and $f(R)$ gravity

A relation to the $f(R)$-gravity theories is established by dropping the gravitino ($\psi^a = 0$) and restricting the auxiliary field B to its real (scalar) component, $B = 3X$ with $\overline{X} = X$. Then the bosonic Lagrangian takes the form

$$L = 2F' \left[\tfrac{1}{3} R + 4X^2 \right] + 6XF \tag{40}$$

It follows that the auxiliary field X obeys an algebraic equation of motion,

$$3F + 11F'X + F'' \left[\tfrac{1}{3} R + 4X^2 \right] = 0 \tag{41}$$

In those equations $F = F(X)$ and the primes denote the derivatives with respect to X. Solving eq. (41) for X and substituting the solution back into eq. (40) results in the bosonic function $f(R)$. The physical sector of the $F(\mathcal{R})$ supergravity is larger than that of the usual supergravity (ie. graviton and gravitino) due to the extra scalar (inflaton), its pseudo-scalar superpartner (axion) and inflatino.

Chaotic inflation in $F(\mathcal{R})$ supergravity (I)

When $F(\mathcal{R}) = f_0 - \frac{1}{2} f_1 \mathcal{R}$ with some (non-vanishing and complex) coefficients f_0 and f_1, one recovers the standard *pure* N=1 Poincaré supergrvity with a negative cosmological term. The relevant term for the slow-roll chaotic inflation in $F(\mathcal{R})$ supergravity is *cubic* in $\mathcal{R}$. We studied the case

$$F(\mathcal{R}) = -\tfrac{1}{2} f_1 \mathcal{R} + \tfrac{1}{2} f_2 \mathcal{R}^2 - \tfrac{1}{6} f_3 \mathcal{R}^3 \qquad (42)$$

whose real coupling constants $f_{1,2,3}$ are of (mass) dimension 2, 1 and 0, respectively. The stability conditions (ie. the absence of ghost and tachyonic degrees of freedom) require $f_1 > 0$ and $f_3 > 0$, whereas the stability of the bosonic embeddding in $F(\mathcal{R})$ supergravity requires $F'(X) < 0$. For the choice (42) the last condition implies $f_2^2 < f_1 f_3$. For simplicity, we used the stronger conditions $f_3 \gg 1$, $f_2^2 \gg f_1$ and $f_2^2 \ll f_1 f_3$. The first one is needed to have inflation at the curvatures much less than $M_{\rm Pl}^2$ (and to meet observations), while the second one is needed to have the scalaron (inflaton) mass be much less than $M_{\rm Pl}$, in order to avoid large (gravitational) quantum loop corrections after the end of inflation up to the present time.

Chaotic inflation in $F(\mathcal{R})$ supergravity (II)

Equation (40) with the Ansatz (42) reads

$$L = -5f_3X^4 + 11f_2X^3 - (7f_1 + \tfrac{1}{3}f_3R)X^2 + \tfrac{2}{3}f_2RX - \tfrac{1}{3}f_1R \tag{43}$$

and gives rise to a cubic equation on X,

$$X^3 - \left(\frac{33f_2}{20f_3}\right)X^2 + \left(\frac{7f_1}{10f_3} + \frac{1}{30}R\right)X - \frac{f_2}{30f_3}R = 0 \tag{44}$$

The *high curvature regime* including inflation is described by

$$\delta R < 0 \quad \text{and} \quad \frac{|\delta R|}{R_0} \gg \left(\frac{f_2^2}{f_1f_3}\right)^{1/3} \tag{45}$$

where we have used the notation $R_0 = 21f_1/f_3 > 0$ and $\delta R = R + R_0$. In the high-curvature regime (45) the f_2-dependent terms in eqs. (43) and (44) can be neglected, and we get

$$X^2 = -\tfrac{1}{30}\delta R \qquad \text{and} \qquad L = -\frac{f_1}{3}R + \frac{f_3}{180}(R + R_0)^2 \tag{46}$$

Chaotic inflation in $F(\mathcal{R})$ supergravity (III)

The value of the coefficient R_0 is not important in the high curvature regime. In fact, it may be changed to a desired value by adding a constant term to the Ansatz (42). Hence, eq. (46) reproduces the Starobinsky inflationary model since inflation occurs at $|R| \gg R_0$. We now identify

$$f_3 = \frac{15}{M^2} \tag{47}$$

The only significant difference with respect to the original $(R + R^2)$ inflationary model is the scalaron mass that becomes much larger than M in supergravity, soon after the end of inflation when δR becomes positive. However, it only makes the scalaron decay faster and creation of the usual matter (reheating) more effective.

The whole series in powers of $\mathcal{R}$ may also be considered, instead of the limited Ansatz (42). The only necessary condition for embedding inflation is that f_3 should be anomalously large.

$F(\mathcal{R})$ supergravity and nonminimal coupling (I)

Let's consider the nonmiminal action (30) under the slow-roll condition, when the contribution of the kinetic term is negligible. Then eq. (30) takes the truly chiral form

$$S_{\text{ch.}} = \int d^4x d^2\theta \mathcal{E} \left[X(\Phi)\mathcal{R} + W(\Phi) \right] + \text{H.c.} \tag{48}$$

When choosing X as the independent chiral superfield, it can be rewritten to

$$S_{\text{ch.}} = \int d^4x d^2\theta \mathcal{E} \left[X\mathcal{R} - \mathcal{Z}(X) \right] + \text{H.c.} \tag{49}$$

where we have introduced the notation

$$\mathcal{Z}(X) = -W(\Phi(X)) \tag{50}$$

In its turn, the action (49) is equivalent to the chiral $F(\mathcal{R})$ supergravity action (38), whose function F is related to the function $\mathcal{Z}$ via Legendre transformation,

$$\mathcal{Z} = X\mathcal{R} - F \ , \quad F'(\mathcal{R}) = X \quad \text{and} \quad \mathcal{Z}'(X) = \mathcal{R} \tag{51}$$

It implies the equivalence between the reduced action (48) and the corresponding $F(\mathcal{R})$ supergravity whose F-function obeys eq. (51).

$F(\mathcal{R})$ supergravity and nonminimal coupling (II)

Consider now the special case of eq. (48) when the superpotential is given by

$$W(\Phi) = \frac{1}{2}m\Phi^2 + \frac{1}{6}\lambda\Phi^3 \tag{52}$$

with the real coupling constants $m > 0$ and $\lambda > 0$. The model (52) is known as the *Wess-Zumino* (WZ) model in 4D, N=1 rigid supersymmetry. It has the most general renormalizable scalar superpotential in the absence of supergravity. In terms of the field components, it gives rise to the Higgs-like scalar potential.

For simplicity, let's take a cubic superpotential,

$$W_3(\Phi) = \frac{1}{6}\lambda\Phi^3 \tag{53}$$

or just assume that this term dominates in the superpotential (52), and choose the $X(\Phi)$-function in eq. (48) in the form

$$X(\Phi) = -\xi\Phi^2 \tag{54}$$

with a large positive coefficient ξ, $\xi > 0$ and $\xi \gg 1$, in accordance with eq. (28).

$F(\mathcal{R})$ supergravity and nonminimal coupling (III)

Let's also simplify the F-function of eq. (42) by keeping only the most relevant cubic term,

$$F_3(\mathcal{R}) = -\frac{1}{6} f_3 \mathcal{R}^3 \tag{55}$$

It is straightforward to calculate the $\mathcal{Z}$-function for the F-function (55) by using eq. (51). We find

$$-X = \frac{1}{2} f_3 \mathcal{R}^2 \quad \text{and} \quad \mathcal{Z}'(X) = \sqrt{\frac{-2X}{f_3}} \tag{56}$$

Integrating the last equation with respect to X yields

$$\mathcal{Z}(X) = -\frac{2}{3}\sqrt{\frac{2}{f_3}}(-X)^{3/2} = -\frac{2\sqrt{2}}{3}\frac{\xi^{3/2}}{f_3^{1/2}}\Phi^3 \tag{57}$$

In accordance to eq. (50), the $F(\mathcal{R})$-supergravity $\mathcal{Z}$-potential (57) implies the superpotential

$$W_{\text{KS}}(\Phi) = \frac{2\sqrt{2}}{3}\frac{\xi^{3/2}}{f_3^{1/2}}\Phi^3 \tag{58}$$

$F(\mathcal{R})$ supergravity and nonminimal coupling (IV)

The derived superpotential (58) coincides with the superpotential (53) of the WZ-model, provided that we identify the couplings as

$$f_3 = \frac{32\xi^3}{\lambda^2} \qquad (59)$$

We thus conclude that the original nonminimally coupled matter-supergravity theory (30) in the slow-roll approximation with the superpotential (53) is classically equivalent to the $F(\mathcal{R})$-supergravity theory with the F-function given by eq. (55) when the couplings are related by eq. (59). The inflaton mass M in the supersymmertic case is given by

$$M^2 = \frac{15\lambda^2}{32\xi^3} \qquad (60)$$

The value of f_3 is of the order $\mathcal{O}(10^{10})$. Therefore, according to eq. (59), the value of ξ in the supersymmetric case is expected to be lower, $\xi \approx \mathcal{O}(10^{10/3})$, when compared to the bosonic case with $\xi \approx \mathcal{O}(10^5)$. We have assumed that λ is of the order one here, $\lambda \approx \mathcal{O}(1)$.

Conclusion (I)

The established equivalence begs for a fundamental reason. In the high-curvature (inflationary) regime the R^2-term dominates over the R-term in the Starobinsky $f(R)$-gravity function (13), while the coupling constant in front of the R^2-action (12) is dimensionless. The Higgs slow-roll inflation is based on the Lagrangian (1), where the $\xi\phi_J^2$ dominates over 1 (in fact, over M_{Pl}^2) in front of the gravitational R-term, and the relevant scalar potential is given by $V_4 = \frac{1}{4}\lambda\phi_J^4$ since the parameter v is irrelevant for inflation, while the coupling constants ξ and λ are also dimensionless. Therefore, both relevant actions are globally conformal. *Inflation spontaneously breaks that conformal symmetry*.

The supersymmetric case is similar: the nonminimal action (48) with the X-function (54) and the superpotential (53) also has only dimensionless coupling constants ξ and λ, while the same it true for the $F(\mathcal{R})$-supergravity action with the F-function (55), whose coupling constant f_3 is dimensionless too. Therefore, those actions are both globally superconformal, while *inflation spontaneously breaks the superconformal invariance*.

Conclusion (II)

A spontaneous breaking of the conformal symmetry, and of the scale invariance, in particular, necessarily leads to Goldstone particle (dilaton) associated with the spontaneously broken scale transformations (dilatations). So, perhaps, the scalaron (inflaton) of Sec. 3 should be *identified* with the Goldstone dilaton related to the spontaneously broken scale invariance (dilatations)!

The equivalence between the non-minimally coupled supergravity and the $F(\mathcal{R})$ supergravity is expected to hold even after inflation, during initial reheating with harmonic oscillations. In the bosonic case the equivalence holds until the inflaton field value is higher than $\omega \approx M_{\rm Pl}/\xi \approx 10^{-5}M_{\rm Pl}$. In the supersymmetric case we find $\omega \approx M_{\rm Pl}/\xi^{3/2} \approx 10^{-5}M_{\rm Pl}$.