Form Factors and Strong Couplings of Heavy Baryons from QCD Light-Cone Sum Rules

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JHEP 09, (2011) 106.

Talk given at DESY Theory Workshop, 28 September, 2011

I. Motivation and introduction

- Weak decays of heavy-baryons are of high interest:
  determination of CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$,
  allow the study of spin correlations (polarization asymmetries...),
  a multitude of new-physics sensitive observables (A_{FB}...).

- Strong coupling constants of (charmed baryon)-(charmed meson)-nucleon are fundamental inputs in the calculations of charm production at $\bar{\text{P}}$ANDA.

- Available techniques to investigate heavy-to-light form factors:
  a)Nonperturbative approaches: Lattice QCD, QCD sum rules, ...
  b)Effective field theories: χPT, HQET, SCET, ...

- Applications of LCSR to the meson transition in mature state:
  $\pi\gamma^* \to \gamma$, $\pi\gamma^* \to \pi$, $B \to \pi \ell \nu_\ell$, $B \to K^* \gamma$, $B \to K^{(*)} \ell^+ \ell^-$, ...

Difficulties of heavy-baryon LCSR

- Background contribution of negative-parity baryon in the dispersion relation:

$$\langle 0|\eta^{(i)}_{\Lambda_c}|\Lambda_c(P-q)\rangle = m_{\Lambda_c}\lambda^{(i)}_{\Lambda_c}\, u_{\Lambda_c}(P-q)\,,$$
$$\langle 0|\eta^{(i)}_{\Lambda_c}|\Lambda^*_c(P-q)\rangle = m_{\Lambda^*_c}\lambda^{(i)}_{\Lambda^*_c}\, u_{\Lambda^*_c}(P-q)\,.$$

  Fermion is not an eigenstate of parity transformation!

- Longstanding issue of interpolating current for baryons:
  Ioffe current or tensor current for nucleon state (Ioffe 1981, Chung et al 1982)?

- Some attempts to avoid background pollution:
  a) Parity projector matrix $(1\pm \not v)/2$ for heavy-baryon sum rule (Bagan et al 1993, ...),
  b)Choose “old-fashioned” correlation function and construct sum rules in the complex q_0-space in the rest frame (Jido et al 1996,...).

Way out in the standard LCSR approach

- A natural scenario to eliminate background pollution using time-ordered correlation function in reference-frame independent way exists?

- How to construct a baryon sum rule with predictions independent on the interpolating current?

- Resolution to the two problems meanwhile:
  a) Separating the negative-parity baryon contribution from continuum,
  b) Constructing two independent LCSR from different kinematical structures,
  c) Choosing a linear combination of two sum rules to remove background pollution.

II. Choices of baryonic currents

- General structure of heavy-baryon current (Shuryak, 1981):

$$\eta = \epsilon^{ijk} \left(q_i \, C \, \Gamma_b \, q'_j \right) \tilde{\Gamma}_b \, Q_k \, .$$

- Isospin symmetry of light diquark system:

$$\left(q \, C \, \Gamma_b \, q' \right)_{\alpha\beta} = (-1)^{I+1} \left(q \, C \, \Gamma_b \, q' \right)_{\beta\alpha} \, .$$

- Three interpolating currents of Λ_Q baryon:

$$\eta^{(\mathcal{P})}_{\Lambda_Q} = (u \, C \, \gamma_5 \, d) \, Q, \qquad \eta^{(\mathcal{A})}_{\Lambda_Q} = (u \, C \, \gamma_5 \gamma_\lambda \, d) \, \gamma^\lambda \, Q \, ,$$

$$\eta^{(\mathcal{S})}_{\Lambda_c} = (u \, C \, d) \, \gamma_5 \, Q \; \text{(Vanishes in the heavy} - \text{quark limit!)}.$$

- Two interpolating currents of Σ_Q baryon:

$$\eta^{(\mathcal{I})}_{\Sigma_c} = (u \, C \, \gamma_\lambda \, d) \, \gamma^\lambda \gamma_5 Q \, , \qquad \eta^{(\mathcal{T})}_{\Sigma_c} = (u \, \sigma_{\mu\nu} \, d) \, \sigma^{\mu\nu} \gamma_5 Q \, .$$

III. LCSR of heavy-baryon form factors

- Definitions of form factors:

$$\langle \Lambda_Q(P-q)|m_Q\bar{Q}\, i\gamma_5\, u|N(P)\rangle = (m_{\Lambda_c}+m_N)G(q^2)\bar{u}_{\Lambda_Q}(P-q)i\gamma_5\, u_N(P)\,,$$

$$\langle \Lambda_Q(P-q)|\bar{Q}\,\gamma_\mu\, u|N(P)\rangle = \bar{u}_{\Lambda_Q}(P-q)\{f_1(q^2)\,\gamma_\mu + i\frac{f_2(q^2)}{m_{\Lambda_Q}}\sigma_{\mu\nu}q^\nu + \frac{f_3(q^2)}{m_{\Lambda_Q}}q_\mu\}u_N(P)\,,$$

$$\langle \Lambda_Q(P-q)|\bar{Q}\,\gamma_\mu\gamma_5\, u|N(P)\rangle = \bar{u}_{\Lambda_Q}(P-q)\{g_1(q^2)\,\gamma_\mu + i\frac{g_2(q^2)}{m_{\Lambda_Q}}\sigma_{\mu\nu}q^\nu + \frac{g_3(q^2)}{m_{\Lambda_Q}}q_\mu\}\gamma_5 u_N(P)\,.$$

- Introducing vacuum-to-nucleon correlation function:

$$\Pi_a(P,q) = i\int d^4z\, e^{iq\cdot z}\langle 0|T\left\{\eta(0), j_a(z)\right\}|N(P)\rangle\,.$$

Weak transition current:

$$j_a = \bar{Q}\,\Gamma_a u\,, \quad \text{with } \Gamma_a = m_Q i\gamma_5, \gamma_\mu, \gamma_\mu\gamma_5\,,$$

- Another corrector with on-shell Λ_Q state and interpolating current for nucleon is also possible!

Hadronic dispersion relation: pseudoscalar transition

- Lorenz decomposition of correlator with EOM:

$$\Pi_5^{(i)}(P,q) = \left[\Pi_1^{(i)}((P-q)^2,q^2) + \not{q}\,\Pi_2^{(i)}((P-q)^2,q^2)\right] i\gamma_5 u_N(P)\,.$$

- Hadronic dispersion relations for invariant amplitudes:

$$\Pi_1^{(i)}((P-q)^2,q^2) = \frac{m_{\Lambda_Q}(m_{\Lambda_Q}^2 - m_N^2)\lambda_{\Lambda_Q}^{(i)} G(q^2)}{m_{\Lambda_Q}^2 - (P-q)^2}$$
$$+\frac{m_{\Lambda_Q^*}(m_{\Lambda_Q^*}^2 - m_N^2)\lambda_{\Lambda_Q^*}^{(i)} \tilde{G}(q^2)}{m_{\Lambda_Q^*}^2 - (P-q)^2} + \int\limits_{s_0^h}^{\infty} ds\, \frac{\rho_1^{(i)}(s,q^2)}{s-(P-q)^2}\,,$$

$$\Pi_2^{(i)}((P-q)^2,q^2) = -\frac{m_{\Lambda_Q}(m_{\Lambda_Q} + m_N)\lambda_{\Lambda_Q}^{(i)} G(q^2)}{m_{\Lambda_Q}^2 - (P-q)^2}$$
$$+\frac{m_{\Lambda_Q^*}(m_{\Lambda_Q^*} - m_N)\lambda_{\Lambda_Q^*}^{(i)} \tilde{G}(q^2)}{m_{\Lambda_Q^*}^2 - (P-q)^2} + \int\limits_{s_0^h}^{\infty} ds\, \frac{\rho_2^{(i)}(s,q^2)}{s-(P-q)^2}\,.$$

Contributions of higher states with the quantum numbers of $\Lambda_Q^{(*)}$ absorbed into $\rho_{1,2}^{(i)}$.

Light-cone sum rules for the form factors

- Light-cone expansion of the correlation function works at space-like region $(P-q)^2,\ q^2 \ll m_Q^2$.

- Generic form of OPE results:

$$\Pi_j^{(i)}((P-q)^2, q^2) \sim \sum_k (T_j^{(i)})_k((P-q)^2, q^2, x) \otimes F_k(x)\,.$$

  Short-distance coefficients T are calculable in perturbative theory.
  Nonperturbative distribution amplitudes of nucleon $F_k(x)$ are universal.

- Light-cone expansion of nonlocal vacuum-to-nucleon matrix element (Braun et al 2001, 2002, ...):

$$\langle 0|\epsilon^{ijk} u_\alpha^i(a_1 z) u_\beta^j(a_2 z) d_\gamma^k(a_3 z)|N(P)\rangle$$
$$= \sum_k \mathcal{F}_k(a_1, a_2, a_3, P\cdot z)\,(\Gamma_k C)_{\alpha\beta}\left(\Gamma'_k u_N\right)_\gamma\,.$$

  27 calligraphic coefficients $\mathcal{F}_k$ emerge up to twist-6 accuracy and can be transformed into LCDAs of the nucleon.

Eliminating negative-parity baryon contribution

- Each form factor enters more than one dispersion relation.

- Making a linear combination of dispersion relations:

$$\frac{m_{\Lambda_Q}(m_{\Lambda_Q}+m_N)(m_{\Lambda_Q}+m_{\Lambda_Q^*})\lambda^{(i)}_{\Lambda_Q} G(q^2)}{m^2_{\Lambda_Q}-(P-q)^2}$$
$$+\int_{s_0^h}^{\infty} ds \, \frac{\rho_1^{(i)}(s,q^2)-(m_{\Lambda_Q^*}+m_N)\rho_2^{(i)}(s,q^2)}{s-(P-q)^2}$$
$$=[\Pi_1^{(i)}((P-q)^2,q^2)-(m_{\Lambda_Q^*}+m_N)\Pi_2^{(i)}((P-q)^2,q^2)]\,.$$

containing only the hadronic matrix elements for the ground-state Λ_Q-baryon!

- Quark-hadron duality:

$$\int_{s_0^h}^{\infty} \frac{ds}{s-(P-q)^2}[\rho_1^{(i)}(s,q^2)-(m_{\Lambda_Q^*}+m_N)\rho_2^{(i)}(s,q^2)]$$

$$=\frac{1}{\pi}\int_{s_0}^{\infty} \frac{ds}{s-(P-q)^2}[\mathrm{Im}_s\Pi_1^{(i)}(s,q^2)-(m_{\Lambda_Q^*}+m_N)\mathrm{Im}_s\Pi_2^{(i)}(s,q^2)]\,.$$

- Borelized sum rules for the form factor:

$$G(q^2)=\frac{e^{m_{\Lambda_Q}^2/M^2}}{m_{\Lambda_Q}(m_{\Lambda_Q}+m_N)(m_{\Lambda_Q}+m_{\Lambda_Q^*})\lambda_{\Lambda_Q}^{(i)}}\frac{1}{\pi}\int_{m_Q^2}^{s_0} ds e^{-s/M^2}$$
$$\times[\mathrm{Im}_s\Pi_1^{(i)}(s,q^2)-(m_{\Lambda_Q^*}+m_N)\mathrm{Im}_s\Pi_2^{(i)}(s,q^2)]\,.$$

- Work out the decay constants $\lambda_{\Lambda_Q^{(i)}}$ with two-point QCD sum rule following the same strategy!

Numerics

- Heavy-baryon decay constants:

$$\lambda_{\Lambda_c}^{(\mathcal{A})} = 1.51^{+0.37}_{-0.39} \times 10^{-2}\ \mathrm{GeV}^2\,, \qquad \lambda_{\Lambda_c}^{(\mathcal{P})} = 1.19^{+0.19}_{-0.28} \times 10^{-2}\ \mathrm{GeV}^2\,,$$
$$\lambda_{\Lambda_b}^{(\mathcal{A})} = 1.27^{+0.35}_{-0.34} \times 10^{-2}\ \mathrm{GeV}^2\,, \qquad \lambda_{\Lambda_b}^{(\mathcal{P})} = 1.09^{+0.31}_{-0.30} \times 10^{-2}\ \mathrm{GeV}^2\,,$$
$$\lambda_{\Sigma_c}^{(\mathcal{I})} = 3.08^{+0.49}_{-0.74} \times 10^{-2}\ \mathrm{GeV}^2\,, \qquad \lambda_{\Sigma_c}^{(\mathcal{T})} = 6.08^{+0.90}_{-1.48} \times 10^{-2}\ \mathrm{GeV}^2\,.$$

- Charm-baryon form factors:

Current Form factor	$\eta_{\Lambda_c}^{(\mathcal{A})}$ ($\Lambda_c \to p$)	$\eta_{\Lambda_c}^{(\mathcal{P})}$ ($\Lambda_c \to p$)	$\eta_{\Sigma_c}^{(\mathcal{I})}$ ($\Sigma_c \to p$)	$\eta_{\Sigma_c}^{(\mathcal{T})}$ ($\Sigma_c \to p$)
$G(0)$	$0.39^{+0.11}_{-0.09}$	$0.48^{+0.13}_{-0.13}$	$0.066^{+0.035}_{-0.032}$	$0.061^{+0.011}_{-0.011}$
$f_1(0)$	$0.46^{+0.15}_{-0.11}$	$0.59^{+0.15}_{-0.16}$	$-0.22^{+0.07}_{-0.07}$	$-0.23^{+0.04}_{-0.05}$
$f_2(0)$	$-0.32^{+0.08}_{-0.07}$	$-0.43^{+0.13}_{-0.12}$	$-0.24^{+0.05}_{-0.05}$	$-0.25^{+0.06}_{-0.06}$
$g_1(0)$	$0.49^{+0.14}_{-0.11}$	$0.55^{+0.14}_{-0.15}$	$0.11^{+0.05}_{-0.05}$	$0.060^{+0.007}_{-0.008}$
$g_2(0)$	$-0.20^{+0.09}_{-0.06}$	$-0.16^{+0.08}_{-0.05}$	$-0.002^{+0.054}_{-0.044}$	$-0.030^{+0.039}_{-0.039}$

- Λ_b-baryon form factors:

form factors	$\eta_{\Lambda_b}^{(\mathcal{A})}$	$\eta_{\Lambda_b}^{(\mathcal{P})}$
$f_1(0)$	$0.14^{+0.03}_{-0.03}$	$0.12^{+0.03}_{-0.04}$
$f_2(0)$	$-0.054^{+0.016}_{-0.013}$	$-0.047^{+0.015}_{-0.013}$
$g_1(0)$	$0.14^{+0.03}_{-0.03}$	$0.12^{+0.03}_{-0.03}$
$g_2(0)$	$-0.028^{+0.012}_{-0.009}$	$-0.016^{+0.007}_{-0.005}$

- LCSR predictions of heavy baryon form factors are insensitive to the heavy-baryon current.

- Symmetry relations in the heavy-quark limit and large-energy limit:

$$f_1(q^2) = g_1(q^2), \qquad f_2(q^2) = g_2(q^2) = f_3(q^2) = g_3(q^2) = 0 .$$

IV. LCSR for the strong couplings

- Definitions of strong coupling constants:

$$\langle \Lambda_c(P-q)|D(-q)N(P)\rangle = g_{\Lambda_c N D}\, \bar{u}_{\Lambda_c}(P-q)\, i\gamma_5\, u_N(P),$$
$$\langle \Lambda_c(P-q)|D^*(-q)N(P)\rangle = \bar{u}_{\Lambda_c}(P-q) \left(g^V_{\Lambda_c N D^*} \not{\epsilon} + i \frac{g^T_{\Lambda_c N D^*}}{m_{\Lambda_c} + m_N} \sigma_{\mu\nu}\epsilon^\mu q^\nu \right) u_N(P).$$

- Heavy-mass relations:

$$g_{\Lambda_c N D} = -g^V_{\Lambda_c N D^*}\,, \qquad g^T_{\Lambda_c N D^*} = 0\,,$$
$$g_{\Sigma_c N D} + 3 g^V_{\Sigma_c N D^*} = \frac{3m_{\Sigma_c} + m_N - 2P\cdot v}{m_{\Sigma_c} + m_N}\, g^T_{\Sigma_c N D^*}\,.$$

- Effective Lagrangian for $\Lambda_c - N - D^{(*)}$ couplings:

$$\mathcal{L}_{\Lambda_c D^{(*)} N} = \bar{\Lambda}_c \left[i a_{\Lambda_c N D} \gamma_5\, D + \left(a^V_{\Lambda_c N D^*} \gamma^\mu + \frac{a^T_{\Lambda_c N D^*}}{m_{\Lambda_c} + m_N} \sigma^{\mu\nu} \partial_\nu \right) D^*_\mu \right] N + h.c.$$

New couplings a_i are generally different from g_i!

LCSR for the strong couplings

- Strong couplings enter double dispersion relations for the same correlation function to construct the sum rule of form factors.

- Hadronic double dispersion relation:

$$
\begin{aligned}
\Pi_5^{(i)}(P,q) &= \frac{\lambda^{(i)}_{\Lambda_c} m_D^2 f_D m_{\Lambda_c} g_{\Lambda_c N D}}{(m^2_{\Lambda_c} - (P-q)^2)(m_D^2 - q^2)} \left[(m_{\Lambda_c} - m_N) - \not{q}\right] i\gamma_5 u_N(P) \\
&+ \frac{\lambda^{(i)}_{\Lambda_c^*} m_D^2 f_D m_{\Lambda_c^*} g_{\Lambda_c^* N D}}{(m^2_{\Lambda_c^*} - (P-q)^2)(m_D^2 - q^2)} \left[(m_{\Lambda_c^*} + m_N) + \not{q}\right] i\gamma_5 u_N(P) \\
&+ \cdots ,
\end{aligned}
$$

- Borelized sum rules for the strong couplings:

$$
\begin{aligned}
g_{\Lambda_c N D} &= \frac{e^{m^2_{\Lambda_c}/M^2} e^{m_D^2/\widetilde{M}^2}}{m_{\Lambda_c}(m_{\Lambda_c} + m_{\Lambda_c^*}) m_D^2 f_D \lambda^{(i)}_{\Lambda_c}} \frac{1}{\pi^2} \int_{m_c^2}^{s_0} ds\, e^{-s/M^2} \\
&\times \int_{t_1(s)}^{t_2(s)} ds'\, e^{-s'/\widetilde{M}^2} \mathrm{Im}_s \, \mathrm{Im}_{s'} [\Pi_1^{(i)}(s,s') - (m_{\Lambda_c^*} + m_N)\Pi_2^{(i)}(s,s')] \, .
\end{aligned}
$$

Numerics

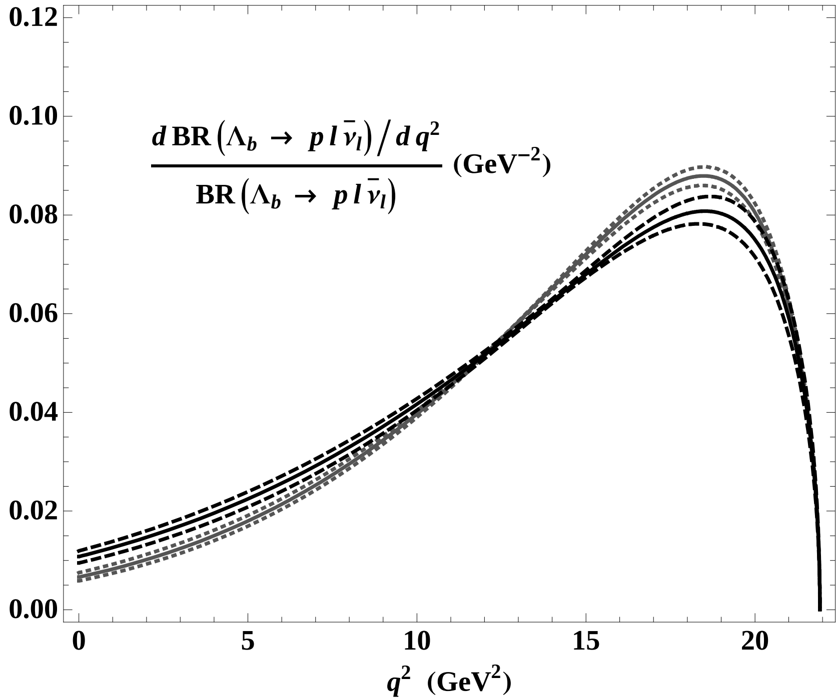
Current Strong couplings	$\eta_{\Lambda_c}^{(\mathcal{A})}$ $\Lambda_c N D^*$	$\eta_{\Lambda_c}^{(\mathcal{P})}$	$\eta_{\Sigma_c}^{(\mathcal{I})}$ $\Sigma_c N D^*$	$\eta_{\Sigma_c}^{(\mathcal{T})}$
$g_{\Lambda_c(\Sigma_c)ND}$	$13.8^{+5.2}_{-4.1}$	$10.7^{+5.3}_{-4.3}$	$1.3^{+1.0}_{-0.9}$	$1.3^{+1.2}_{-0.8}$
$g^V_{\Lambda_c(\Sigma_c)ND^*}$	$-7.9^{+2.7}_{-3.3}$	$-5.8^{+2.1}_{-2.5}$	$1.0^{+1.3}_{-0.6}$	$0.74^{+1.08}_{-0.45}$
$g^T_{\Lambda_c(\Sigma_c)ND^*}$	$4.7^{+2.7}_{-2.0}$	$3.6^{+2.9}_{-1.8}$	$2.1^{+1.9}_{-1.0}$	$1.8^{+1.6}_{-0.8}$

- LCSR predictions of strong coupling constants are insensitive to the heavy-baryon current.
- The heavy-mass relations for the three strong couplings of Λ_c baryon are only qualitatively supported by the LCSR predictions.
- The results for $\Sigma_c N D^{(*)}$ couplings are in good agreement with the heavy mass relation.

V. Applications to exclusive Λ_b decays

- Apply the conformal mapping $q^2 \to z$ and z-series parametrization to extrapolate the form factors to the whole semileptonic $\Lambda_b \to pl\nu$ region.

- Normalized differential width of $\Lambda_b \to p\ell\nu_\ell$:



The enhancement in the region of large q^2 due to the growth of the form factors and the S-wave phase-space factor $\lambda^{1/2}$.

- Total branching fraction:

$$BR(\Lambda_b \to p l \nu_l) = \left\{ \begin{array}{l} \left(3.3^{+1.5}_{-1.2}|_{th.} \pm 0.1|_{exp.}\right) \\ \left(4.0^{+2.3}_{-2.0}|_{th.} \pm 0.1|_{exp.}\right) \end{array} \right\} \left(\frac{|V_{ub}|}{3.5 \cdot 10^{-3}}\right)^2 \times 10^{-4},$$

form factors from LCSR with axial-vector (pseudoscalar) Λ_b current.

About three times of $BR(B^0 \to \pi^- l^+ \nu_l) = (1.41 \pm 0.05 \pm 0.07) \times 10^{-4}$!

- Branching ratio in factorization limit:

$$\mathrm{BR}(\Lambda_b \to p\pi) = 3.8^{+1.3}_{-1.0} \left(2.8^{+1.1}_{-0.9}\right) \times 10^{-6},$$

obtained with the axial-vector (pseudoscalar) Λ_b interpolating current.

Agree with experimental measurement (CDF, 2009):
$\mathrm{BR}(\Lambda_b \to p\pi) = (3.5 \pm 0.6 \pm 0.9) \times 10^{-6}$!

Summary

- Heavy baryon form factors and strong couplings are calculated in QCD light-cone sum rule avoiding background pollution.
- Our predictions are less sensitive to the particular choice of baryon currents.
- Heavy-mass relations of form factors and strong couplings are respected by explicit LCSR calculations.
- Differential (integrated) decay width of semileptonic $\Lambda_b \to p l \nu$ predicted. Potential way to determine $|V_{ub}|$.
- Nonleptonic $\Lambda_b \to p \pi$ decay computed in the factorization limit is consistent with CDF measurement.
- More applications to charm production will appear soon!