ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Cosmological Inflation and the Standard Model

Mikhail Shaposhnikov

Cosmology meets Particle Physics:

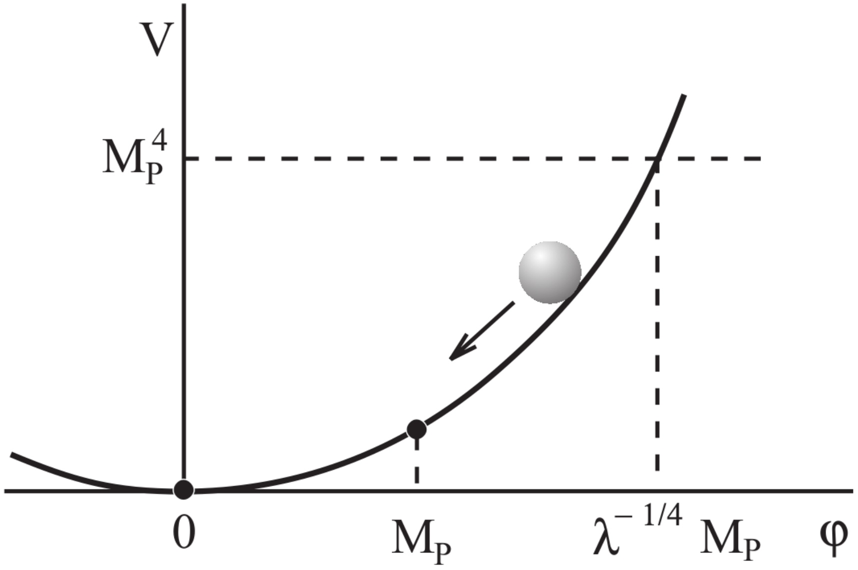
Ideas & Measurements

DESY, 27 - 30 September 2011

Outline

- Inflation: new dynamics versus new particle
- Standard Model Higgs boson as inflaton
- CMB parameters in Higgs inflation
- Higgs mass from inflation
- Comparison with other works
- Naturalness of Higgs inflation
- Scale invariance, inflation and Dark Energy
- Conclusions

“Standard” chaotic inflation



Required for inflation: (to get $\delta T/T \sim 10^{-5}$)

- quartic coupling constant $\lambda \sim 10^{-13}$
- mass $m \sim 10^{13}$ GeV,

Present in the Standard Model:

Higgs boson

- $\lambda \sim 1$, $m_H \sim 100$ GeV
- $\delta T/T \sim 1$

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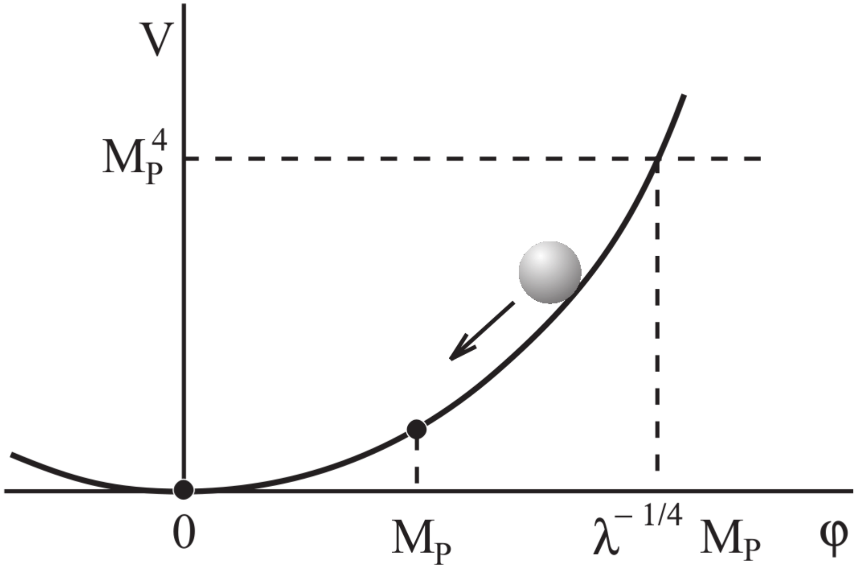

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New physics is required?

No - these conclusions are based on a theory with minimal coupling of scalar to gravity!:

$$S = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R + g_{\mu\nu}\frac{\partial^\mu h \partial^\nu h}{2} - \frac{\lambda}{4}\left(h^2 - v^2\right)^2\right\}$$

Extra term, necessary for renormalizability:

non-minimal coupling of scalar to gravity

$$\Delta S = \int d^4x\sqrt{-g}\left\{-\frac{\xi h^2}{2}R\right\}$$

Feynman, Brans, Dicke,...

Standard Model Higgs boson as inflaton

Bezrukov, M.S.: New dynamics versus new particle

- Gravity strength: $M_P^{\text{eff}} = \sqrt{M_P^2 + \xi h^2} \propto h$
- All particle masses are $\propto h$

For $h > \frac{M_P}{\xi}$ (classical) physics is the same (M_W / M_P^{eff} does not depend on h)!

Existence of effective flat direction, necessary for successful inflation.

Formalism: go from Jordan frame to Einstein frame with the use of conformal transformation:

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \; , \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

Redefinition of the Higgs field to make canonical kinetic term

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi_h^2 h^2/M_P^2}{\Omega^4}} \quad \Longrightarrow \quad \begin{cases} h \simeq \chi & \text{for } h < M_P/\xi \\ h \simeq \frac{M_P}{\sqrt{\xi}} \exp\left(\frac{\chi}{\sqrt{6}M_P}\right) & \text{for } h > M_P/\sqrt{\xi} \end{cases}$$

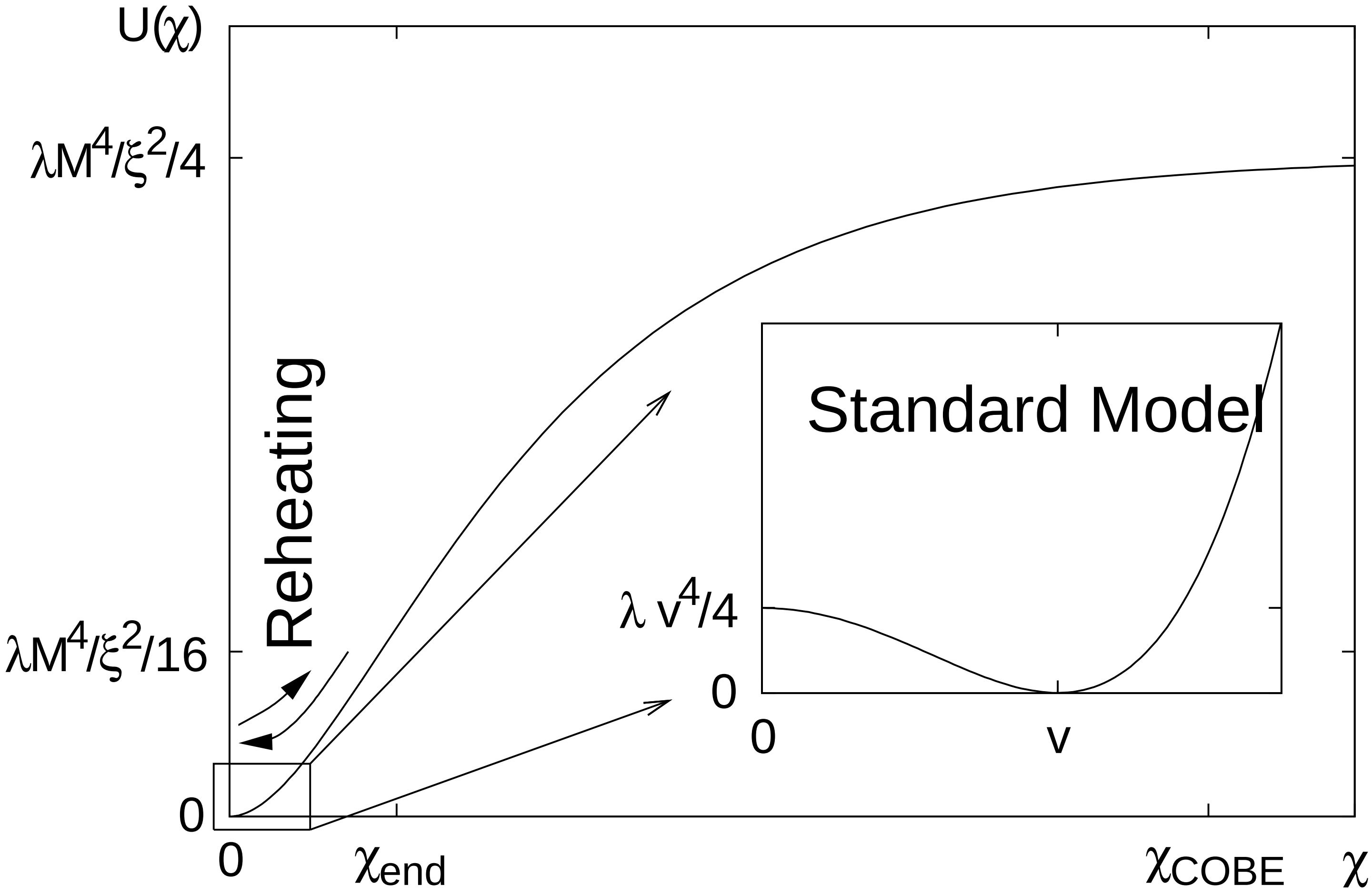
Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - \frac{1}{\Omega(\chi)^4}\frac{\lambda}{4}h(\chi)^4 \right\}$$

Potential:

$$U(\chi) = \begin{cases} \frac{\lambda}{4}\chi^4 & \text{for } h < M_P/\xi \\ \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2 & \text{for } h > M_P/\xi \end{cases} .$$

Potential in Einstein frame



Inflaton potential and observations

If inflaton potential is known one can make predictions and compare them with observations.

- $\delta T/T$ at the WMAP normalization scale ~ 500 Mpc
- The value of spectral index n_s of scalar density perturbations

$$\left\langle \frac{\delta T(x)}{T}\frac{\delta T(y)}{T}\right\rangle \propto \int \frac{d^3k}{k^3} e^{ik(x-y)} k^{n_s-1}$$

- The amplitude of tensor perturbations $r = \frac{\delta\rho_s}{\delta\rho_t}$

These numbers can be extracted from WMAP observations of cosmic microwave background. Higgs inflation: one new parameter, $\xi \Longrightarrow$ two predictions.

Slow roll stage

COBE normalization $U/\epsilon = (0.027 M_P)^4$ gives

$$\xi \simeq \sqrt{\frac{\lambda}{3}\frac{N_{\rm COBE}}{0.027^2}} \simeq 49000\sqrt{\lambda} = 49000\frac{m_H}{\sqrt{2}v}$$

Connection of ξ and the Higgs mass!

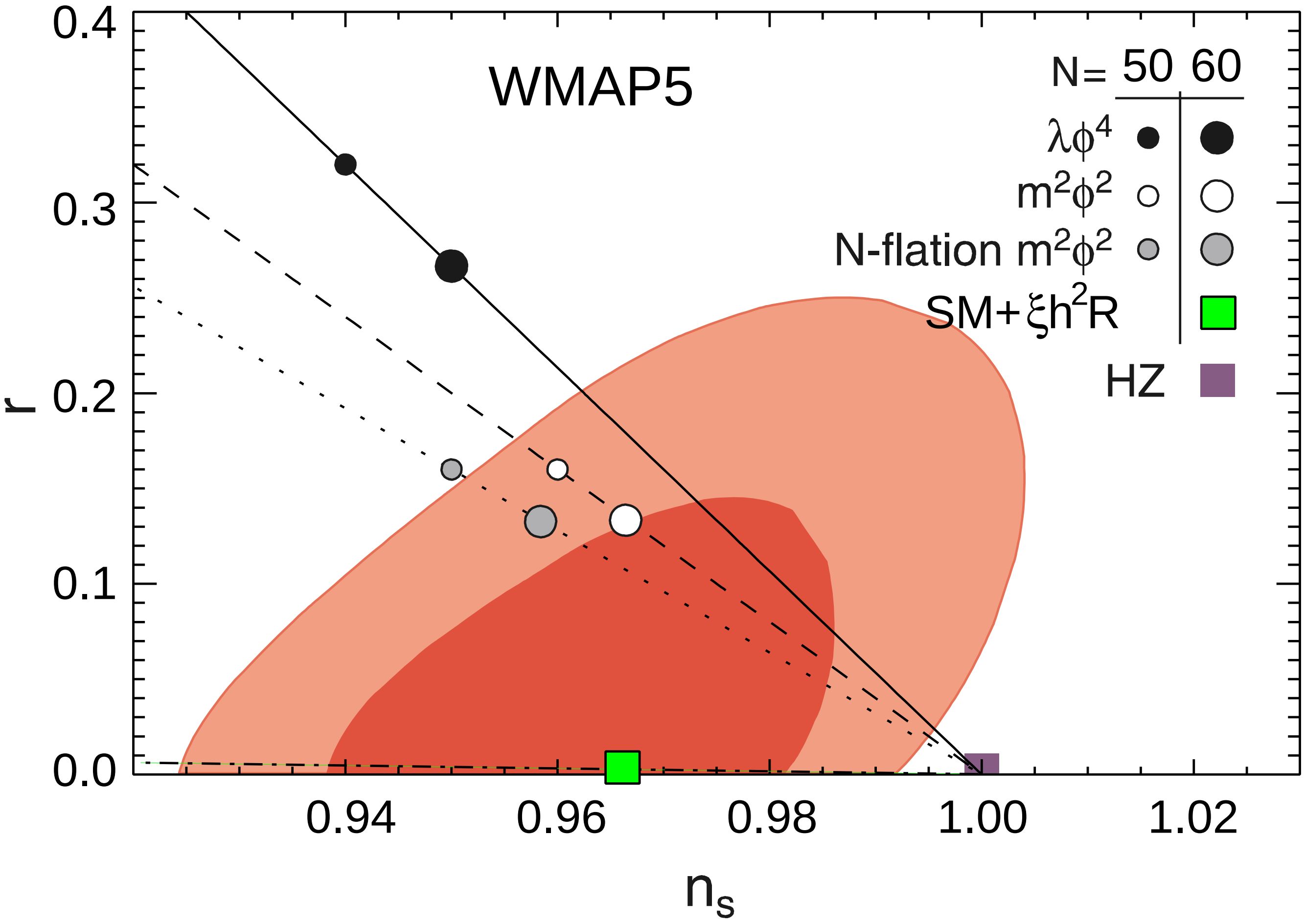
Number of e-folds of inflation at the moment h_N is $N \simeq \frac{6}{8}\frac{h_N^2 - h_{\rm end}^2}{M_P^2/\xi}$

Slow roll ends at $\chi_{\rm end} \simeq M_P$; and “begins” at $\chi_{60} \simeq 5M_P$

$$\epsilon = \frac{M_P^2}{2}\left(\frac{dU/d\chi}{U}\right)^2, \quad \eta = M_P^2\frac{d^2U/d\chi^2}{U}$$

$$n_s = 1 - 6\epsilon + 2\eta, \qquad r = 16\epsilon$$

CMB parameters—spectrum and tensor modes



Earlier works: non-minimal coupling of scalars in GUTs, etc:

- B. Spokoiny ’84
- D. Salopek, J. Bond and J. Bardeen ’89
- R. Fakir and W. G. Unruh ’90
- A. O. Barvinsky and A. Y. Kamenshchik ’94, ’98
- E. Komatsu and T. Futamase ’99
- S. Tsujikawa and B. Gumjudpai ’04

Computation of spectral indexes gives the same results in Einstein and Jordan frames.

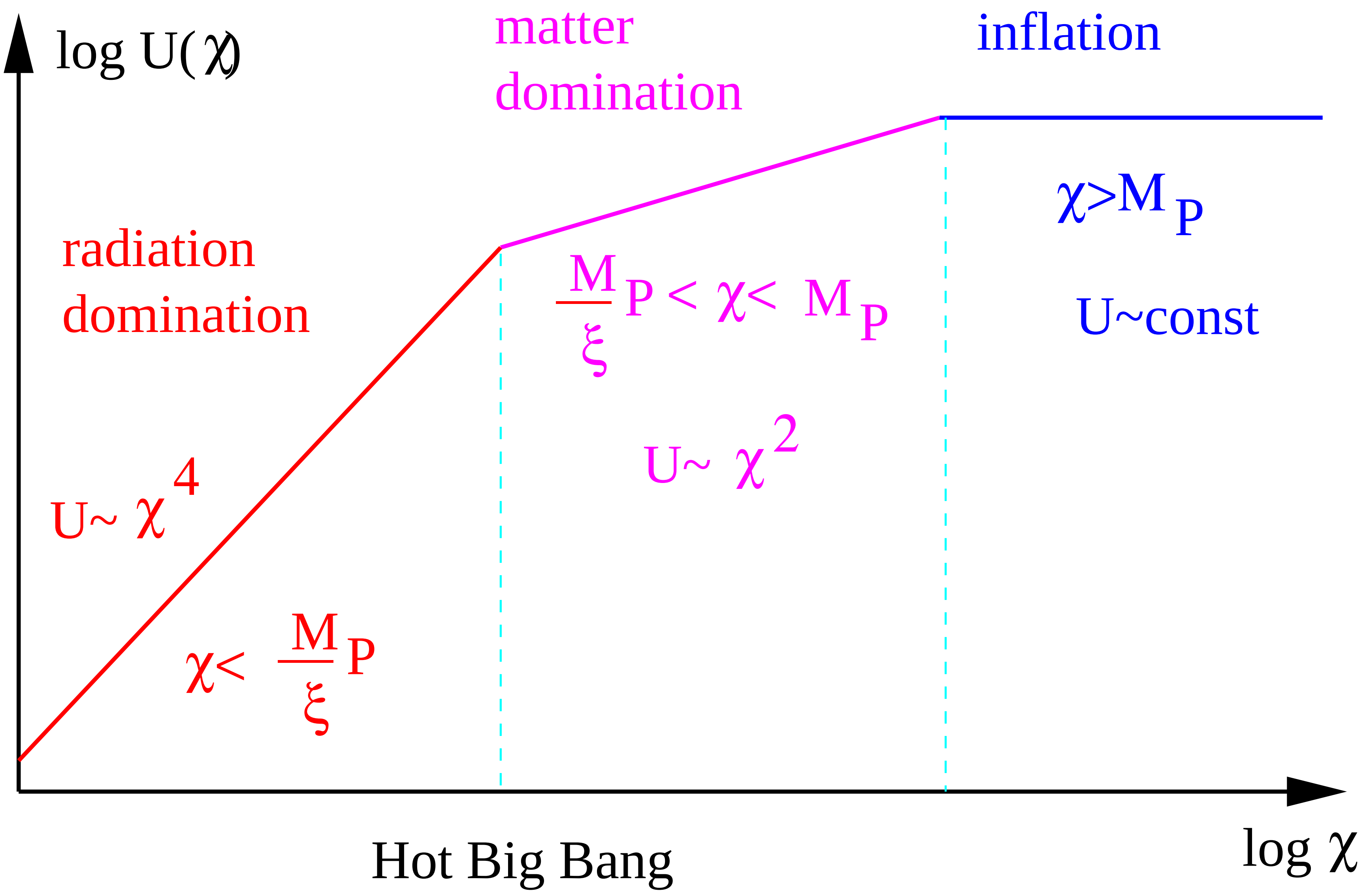
Life after inflation

Two different stages:

- For scalar field $M_P > \chi > \frac{M_P}{\xi}$ the potential for χ is essentially *quadratic*, $m_\chi^2 \sim \lambda M_P^2/\xi^2$. Exponential expansion of the Universe is changed to the power low, corresponding to matter domination. Particle creation takes place when χ passes through zero.
- After $\mathcal{O}(\xi)$ oscillations the scalar field reaches $\chi \simeq \frac{M_P}{\xi}$. The energy is transferred to other fields of the SM, and the radiation-dominated epoch starts,

$$T_r \simeq (3.3 - 8.3) \times 10^{13}\ \mathrm{GeV}.$$

Bezrukov, Gorbunov, M.S.; J. Garcia-Bellido, D. G. Figueroa, J. Rubio



log $U(\chi)$

radiation domination

$U \sim \chi^4$

$\chi < \frac{M_P}{\xi}$

matter domination

$\frac{M_P}{\xi} < \chi < M_P$

$U \sim \chi^2$

inflation

$\chi > M_P$

$U \sim \text{const}$

Hot Big Bang

log χ

Higgs mass from inflation: qualitative argument

Previous consideration tells nothing about the Higgs mass: change λ as $\propto \xi^2$ - no modifications!

However: λ is not a constant, it depends on the energy. Typical scale at inflation $\sim M_P/\sqrt{\xi}$.

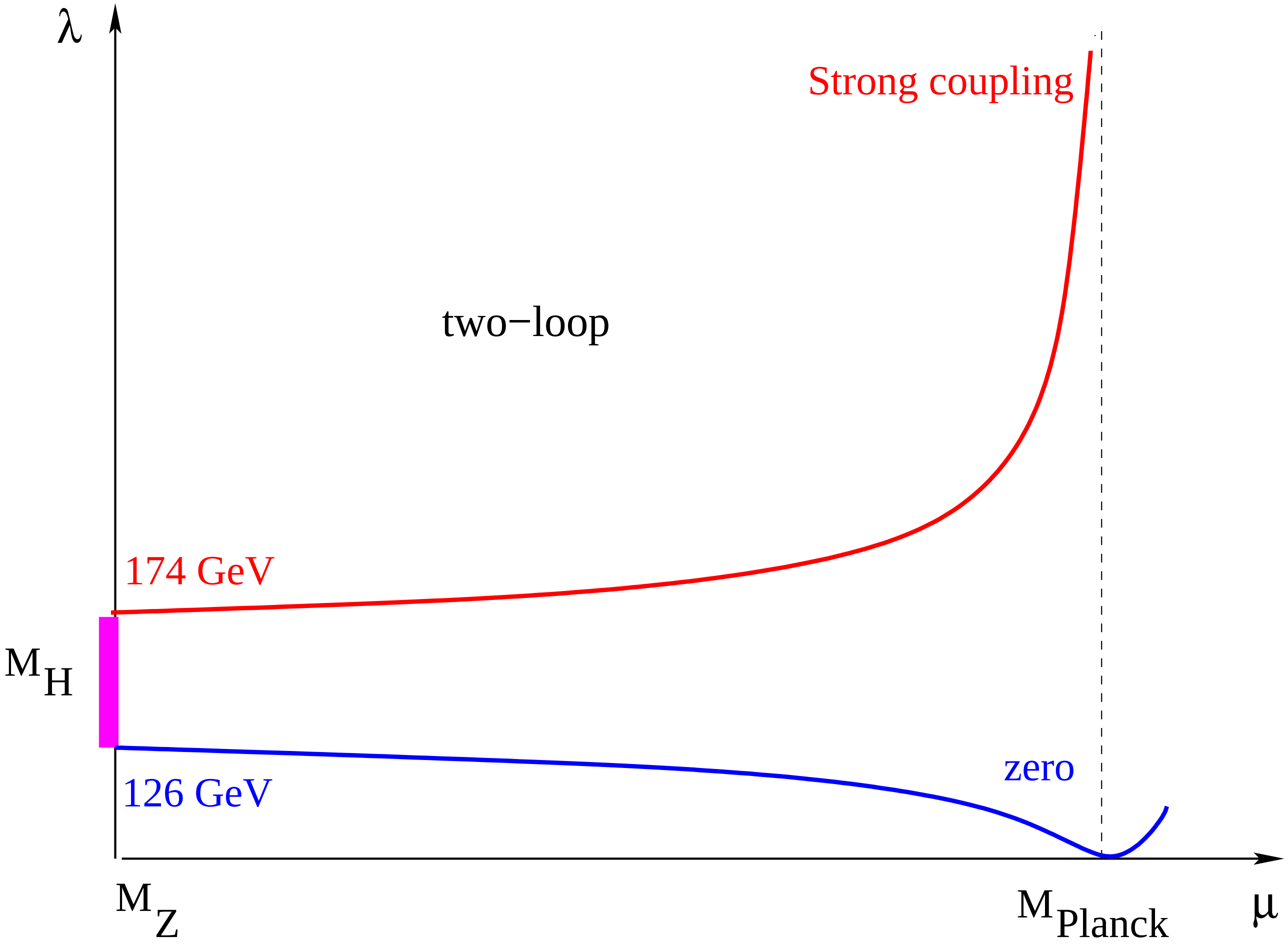
Therefore, SM must be a valid quantum field theory up to the Inflation (or, to be on safe side, up to the Planck scale).

$$m_{\text{min}} < m_H < m_{\text{max}}$$

$$m_{\text{min}} = [129.5 + \frac{m_t - 173.1}{2.1} \times 4.1 - \frac{\alpha_s - 0.1183}{0.002} \times 0.6]\ \text{GeV}$$

$$m_{\text{max}} = [174.0 + \frac{m_t - 173.1}{2.1} \times 0.6 - \frac{\alpha_s - 0.1183}{0.002} \times 0.1]\ \text{GeV}$$

Behaviour of the scalar self-coupling



For $m_H > m_{\rm max}$: Landau pole for energies $E = E_{\rm Landau} < M_P$ – quantum field theory is inconsistent for $E > E_{\rm Landau}$.

For $m_H < m_{\rm min}$: Electroweak vacuum is unstable: there is a lower ground state at $\phi < M_P$.

Higgs mass from inflation: computation

Electroweak theory in the inflationary region, for

$h \sim M_P/\sqrt{\xi}, \;\; h \gg M_P/\xi$:

Take the SM, freeze the radial mode of the Higgs field, and add to Lagrangian almost massless and almost non-interacting scalar: chiral SM.

Why the Higgs decouples?

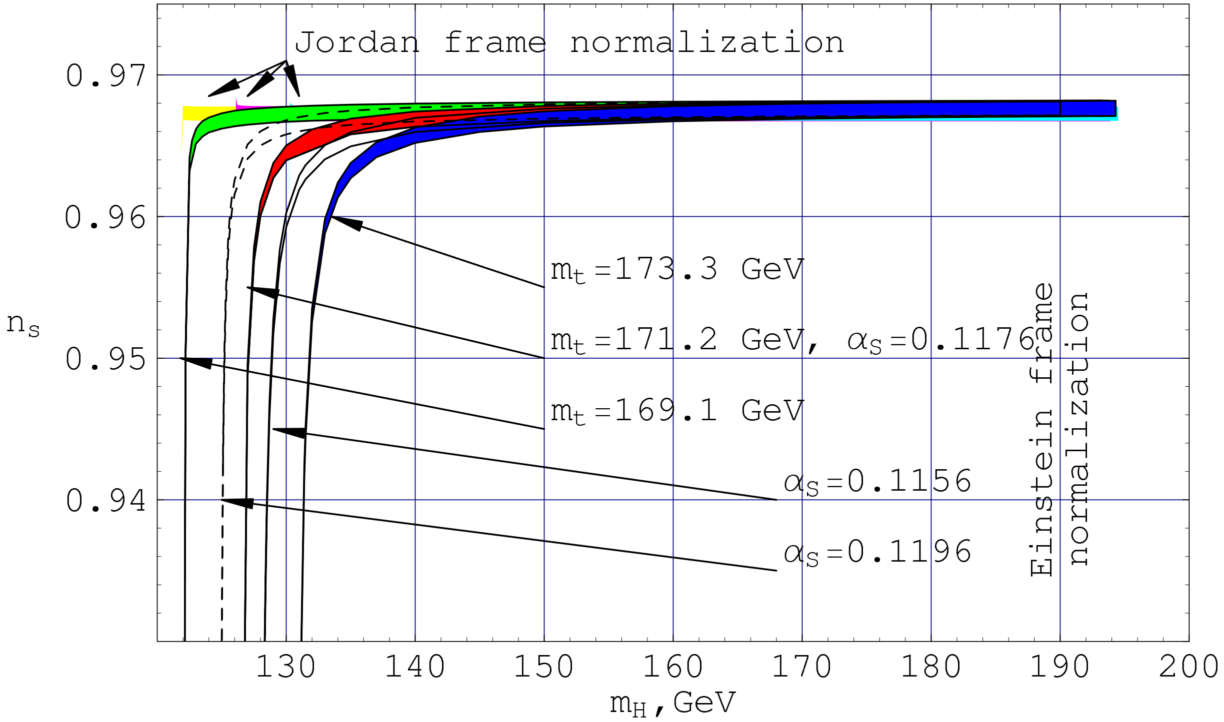
Einstein frame: masses of all the particles

$$M_W, \ldots, m_t \propto v = \frac{h}{\Omega(h)} \to \text{const for } h \to \infty$$

The procedure for computations of inflationary parameters

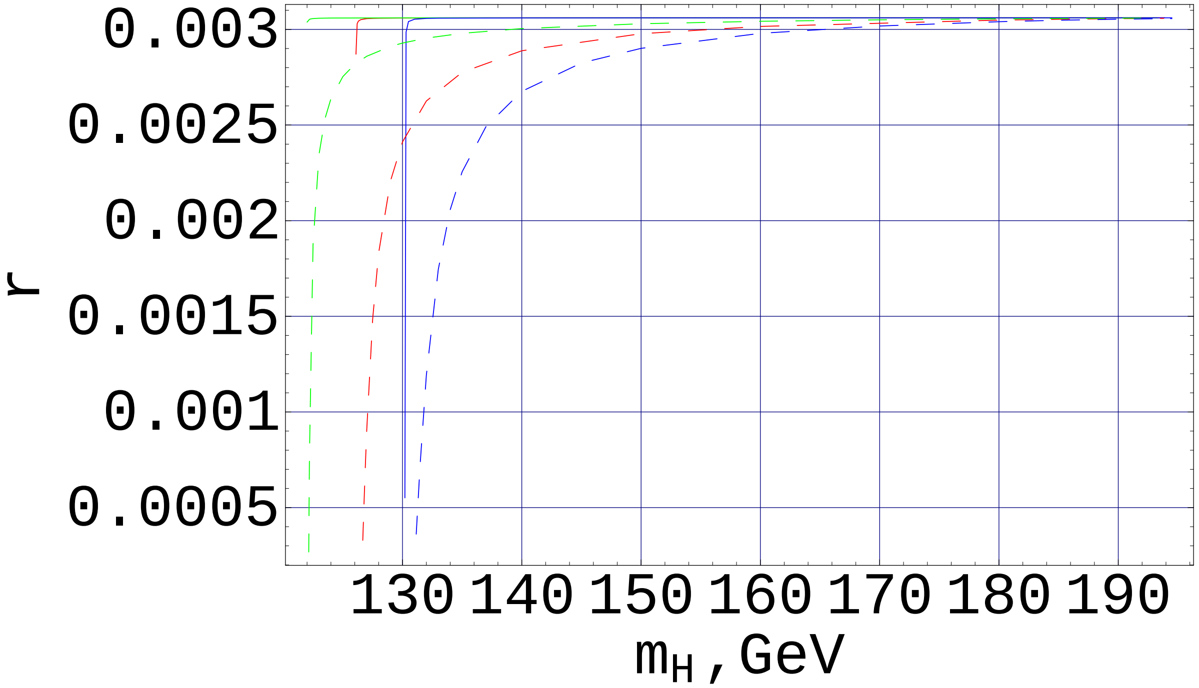
- Compute the effective potential in the inflationary region (tree, one-loop, two-loop,... with the use of the chiral SM
- Choose the normalization point μ to minimize higher order terms (normally, $\mu \sim M_W$ or M_Z or m_t)
- To find values of different coupling constants in inflationary region, solve one-loop, two-loop ... RG equations, getting initial conditions from tree, one-loop, two-loop,.. relations mapping physical SM parameters to couplings.
- Find ξ from COBE normalization and compute n_s and r as a function of the Higgs mass

Two-loop results



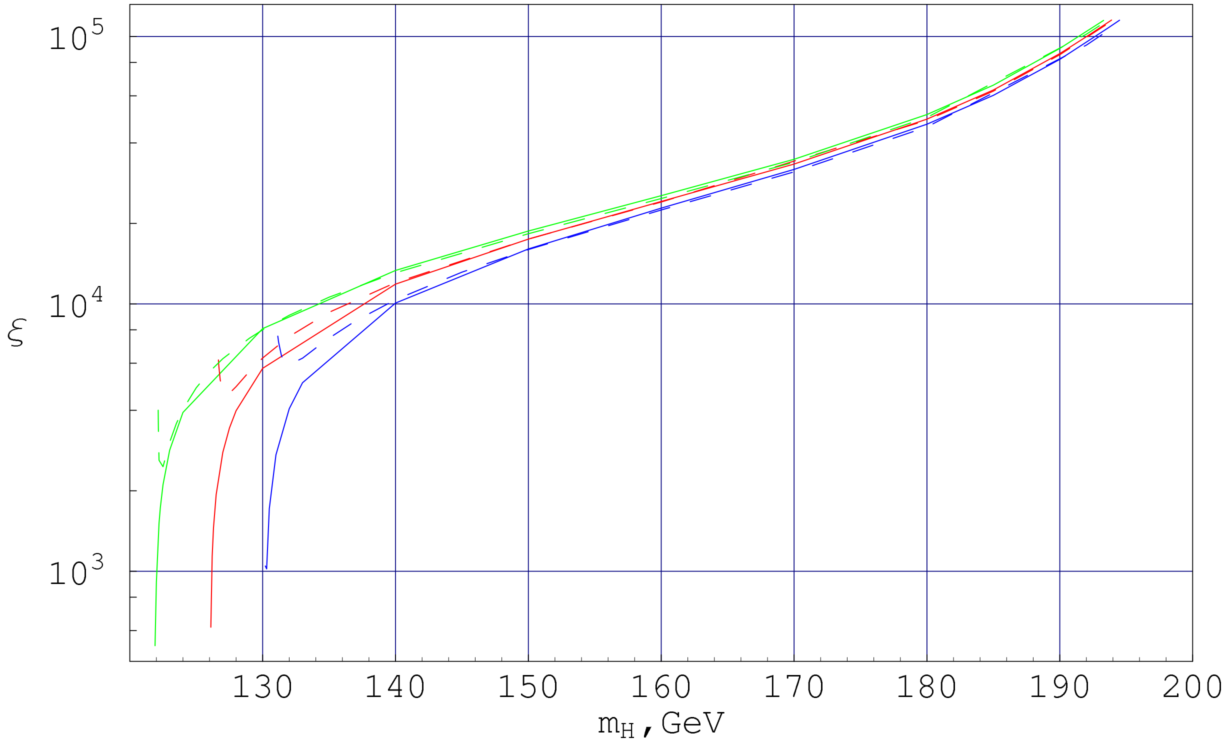
Nearly horizontal coloured stripes correspond to the normalization prescription I. Green, red, and blue stripes give the result with normalization prescription II for different m_t and $\alpha_s = 0.1176$, two white regions correspond to different α_s and $m_t = 171.2\text{GeV}$. The width of the stripes corresponds to changing the number of e-foldings between 58 and 60, or approximately one order of magnitude in reheating temperature.

Two-loop results



Tensor-to-scalar ratio r depending on the Higgs mass m_H, calculated with the RG enhanced effective potential. Nearly horizontal solid lines correspond to the normalization prescription I. Green, red, and blue dashed lines give the result with normalization prescription II for $m_t = 169.1, 171.2, 173.3$ GeV. Dependence on the number of e-foldings is very small.

Two-loop results



$\boldsymbol{\xi}$ at the scale $\boldsymbol{M_P/\xi}$ depending on the Higgs mass $\boldsymbol{m_H}$ for $\boldsymbol{m_t = 171.2, 169.1, 173.3}$ **GeV** (from upper to lower graph). Solid lines correspond to prescription I, dashed—to prescription II. Changing the e-foldings number and error in the WMAP normalization measurement introduce changes invisible on the graph.

Cosmological constraint on the Higgs mass

1 loop computation

$$m_{\min} = [140.4 + (m_t - 173.1) \times 1.95]\ \mathrm{GeV}$$
$$m_{\max} = [185.5 + (m_t - 173.1) \times 0.5\ \ ]\ \mathrm{GeV}$$

2 loop computation

$$m_{\min} = [129.3 + \frac{m_t - 173.1}{2.1} \times 4.1 - \frac{\alpha_s - 0.1183}{0.002} \times 1.5]\ \text{GeV}\ ,$$
$$m_{\max} = [194.4 + \frac{m_t - 173.1}{2.1} \times 0.6 - \frac{\alpha_s - 0.1183}{0.002} \times 0.1]\ \text{GeV}\ .$$

Main effect - the window for the Higgs mass is wider. Theoretical uncertainty ± 2.2 GeV. Spectral index behaviour is the same in one and two loops.

Experimental constraints on the Higgs mass

Direct searches

LEP limit: $m_H > 114.4$ GeV, 95% C.L.

LHC limit: $m_H < 145$ GeV, 95% C.L.

Higgs inflation works roughly in half of the admitted region

Wetterich, M.S.: If gravity is asymptotically safe and gravity contribution to the anomalous dimension of scalar coupling is positive, then lower point of admitted region is selected:

$$M_H = 129.5 \pm 2.7(exp) \pm 2.2(theor)\ \mathrm{GeV}$$

Comparison with other works

- Result of A. Barvinsky, A. Kamenshchik, C. Kiefer, A. Starobinsky and C. Steinwachs (one loop) : Higgs inflation can take place if

$$135.6 \text{ GeV} < m_H < 184.5 \text{ GeV}$$

Their 1-loop numbers agree with our 1-loop numbers

- Result of A. De Simone, M. Hertzberg and F. Wilczek :
  Higgs inflation works if

$$m > [129.2 + \frac{m_t - 173.1}{2} \times 3.8 - \frac{\alpha_s - 0.1183}{0.0020} \times 1.4] \text{ GeV}$$

Our 2-loop computation of $m_{\rm min}$ is in good agreement with their work.

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If ξ is large then chaotic inflation is inevitable in the Standard model, $V_{\text{inf}} \propto \lambda M_P^4/\xi^2$.

What happens at large ξ?

Sibiryakov, ’08; Burgess, Lee, Trott, ’09; Barbon and Espinosa, ’09

Tree amplitudes of scattering of scalars above electroweak vacuum hit the unitarity bound at energies

$$E > \Lambda \sim \frac{M_P}{\xi}$$

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What does it mean?

Option 1: The theory fails and must be replaced by a more fundamental one

Option 2: A theorist fails and must work harder to figure out what happens

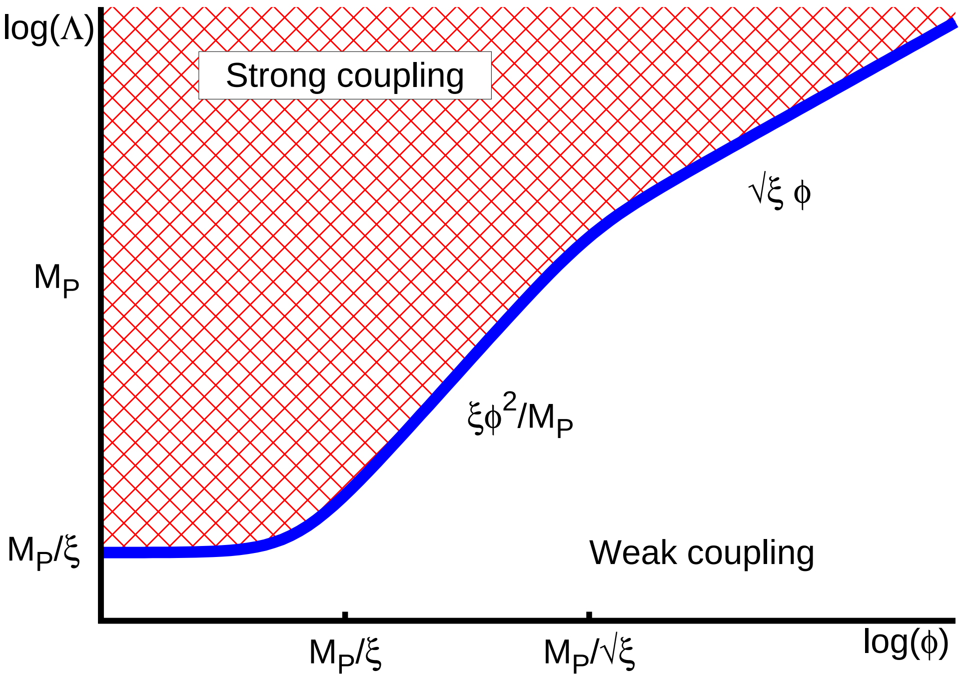
Effective theory

We do not know the more fundamental theory. So, lets add to the SM all sorts of higher dimensional operators suppressed by powers of cutoff Λ. Cutoff is background dependent: Bezrukov, Magnin, M.S., Sibiryakov; Ferrara, Kallosh, Linde, A. Marrani, Van Proeyen

$$\Lambda(h) \simeq \begin{cases} \frac{M_P}{\xi}\ , & \text{for } h \lesssim \frac{M_P}{\xi}\ , \\ \frac{h^2\xi}{M_P}\ , & \text{for } \frac{M_P}{\xi} \lesssim h \lesssim \frac{M_P}{\sqrt{\xi}}\ , \\ \sqrt{\xi}h\ , & \text{for } h \gtrsim \frac{M_P}{\sqrt{\xi}}\ . \end{cases}$$

Important: scale invariance in Jordan frame = shift symmetry in Einstein frame

Higgs-dependent cutoff



Cutoff is higher than the relevant dynamical scales throughout the whole history of the Universe, including the inflationary epoch and reheating!!

The Higgs-inflation is “natural” in the Standard Model.

Scale invariance, inflation and Dark Energy

Essential condition for Higgs inflation - approximate scale invariance at asymptotic values of the Higgs field.

Conjecture: the fundamental theory, including (unimodular) gravity, has exact, but spontaneously broken scale invariance.

Zenhausern, M.S.

Consequences

- Dynamical origin of all mass scales
- Hierarchy problem gets a different meaning - an alternative (to SUSY, techicolor, little Higgs or large extra dimensions) solution of it may be possible
- Cosmological constant problem acquires another formulation
- Natural chaotic cosmological inflation
- Low energy sector contains a massless dilaton
- There is Dark Energy even without cosmological constant. It is related to the dilaton and to new conservation law specific to unimodular gravity

Realisation

Unique regular scale-invariant Lagrangian

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm SM[M\to 0]} + \mathcal{L}_G + \frac{1}{2}(\partial_\mu \chi)^2 - V(\varphi, \chi)$$

Potential (χ - dilaton, φ - Higgs, $\varphi^\dagger\varphi = 2h^2$):

$$V(\varphi,\chi) = \lambda\left(\varphi^\dagger\varphi - \frac{\alpha}{2\lambda}\chi^2\right)^2 + \beta\chi^4,$$

Gravity part, $det[g] = -1$

$$\mathcal{L}_G = -\left(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger\varphi\right)\frac{R}{2}\,,$$

For $\lambda > 0, \quad \beta = 0$ the scale invariance can be spontaneously broken.

The vacuum manifold:

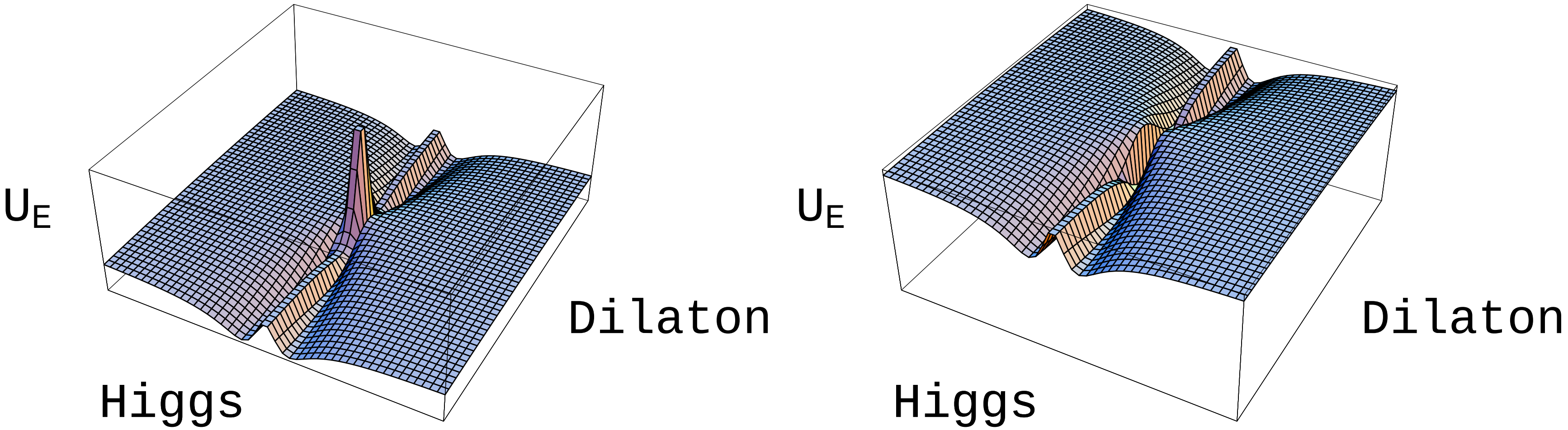
$$h_0^2 = \frac{\alpha}{\lambda}\chi_0^2$$

Particles are massive, Planck constant is non-zero:

$$M_H^2 \sim M_W \sim M_t \sim M_N \propto \chi_0, \quad M_{Pl} \sim \chi_0$$

Phenomenological requirement:

$$\alpha \sim \frac{v^2}{M_{Pl}^2} \sim 10^{-38} \lll 1$$



Potential for the Higgs field and dilaton in the Einstein frame.

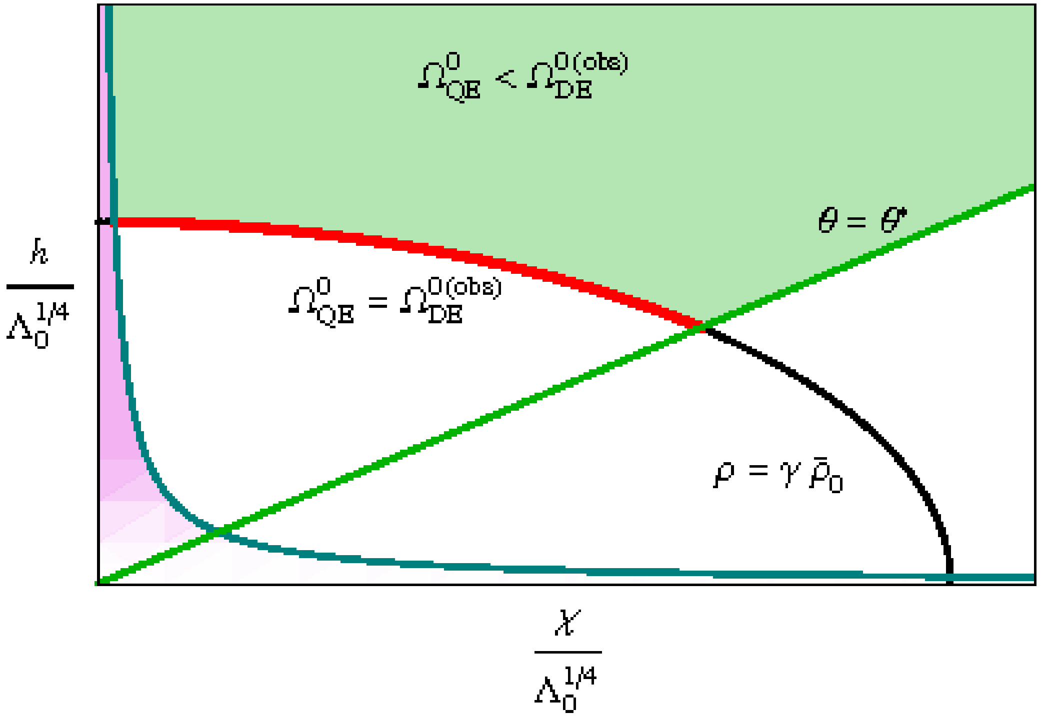
Left: $\boldsymbol{\Lambda > 0}$, right $\boldsymbol{\Lambda < 0}$.

$\mathbf{50\%}$ chance ($\boldsymbol{\Lambda < 0}$): inflation + late collapse

$\mathbf{50\%}$ chance ($\boldsymbol{\Lambda > 0}$): inflation + late acceleration

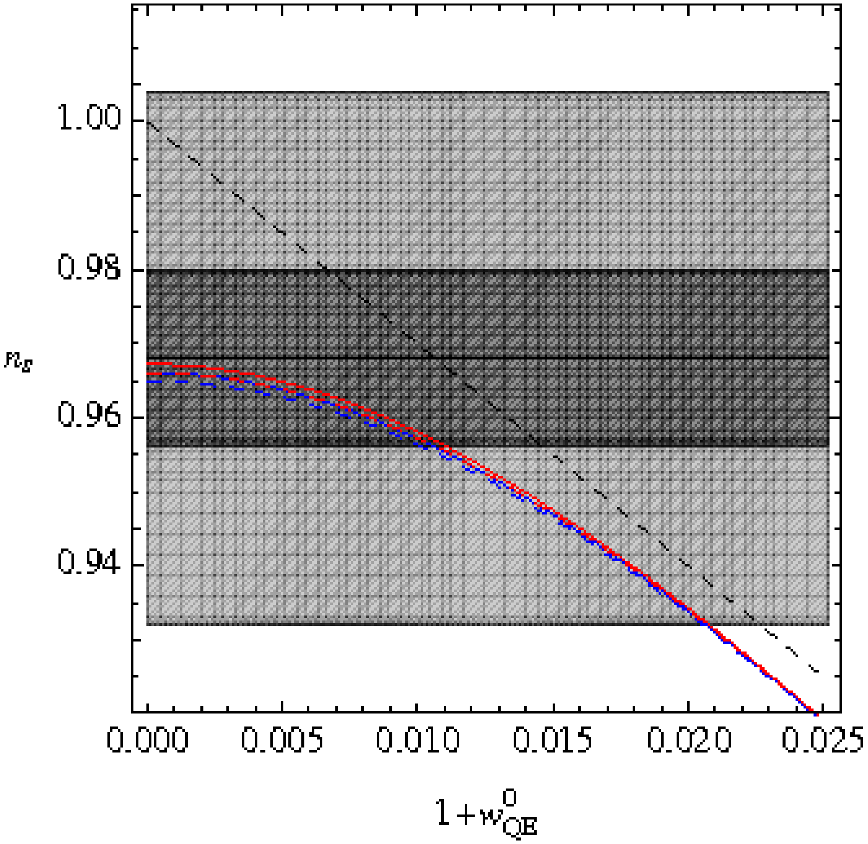
Initial conditions

Juan García-Bellido, Javier Rubio, M.S., Daniel Zenhäusern



Inflation-dark energy relation

Juan García-Bellido, Javier Rubio, M.S., Daniel Zenhäusern



ω - equation of state of DE

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- This region is somewhat wider that the region of validity of the SM all the way up to the Planck scale $M_H \in [129.5, 174]$ GeV (for central values of m_t and α_s).

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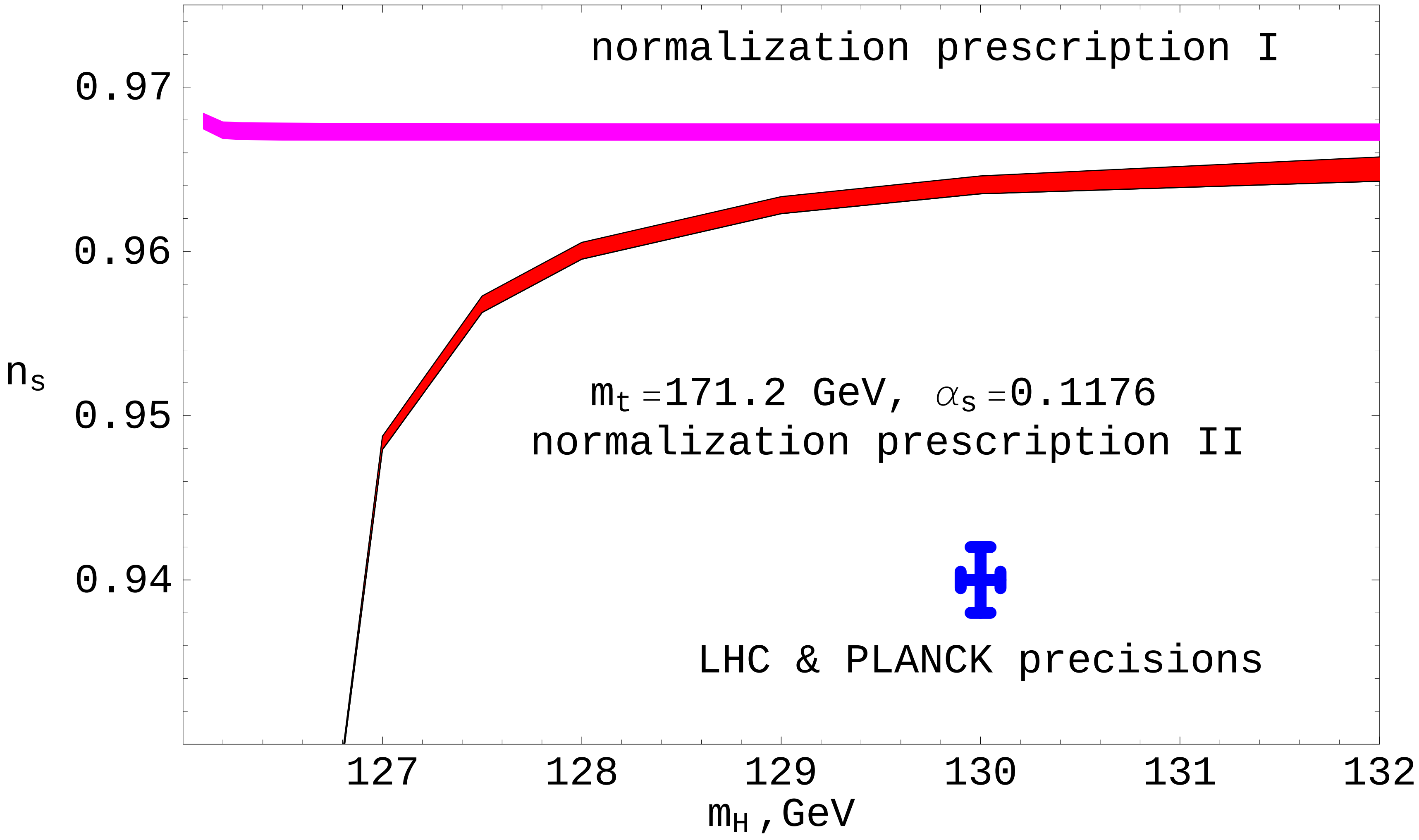
- Crucial experimental test - the LHC: nothing but the Higgs boson
- Crucial cosmological test - precise measurements of cosmological parameters $n_s, \;\; r, \;\; \omega_{DE}$

Experimental precision



Open problems

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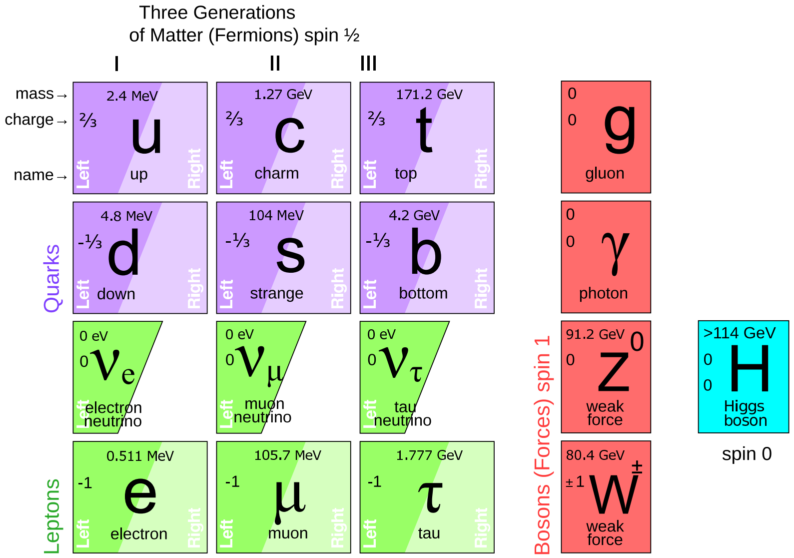
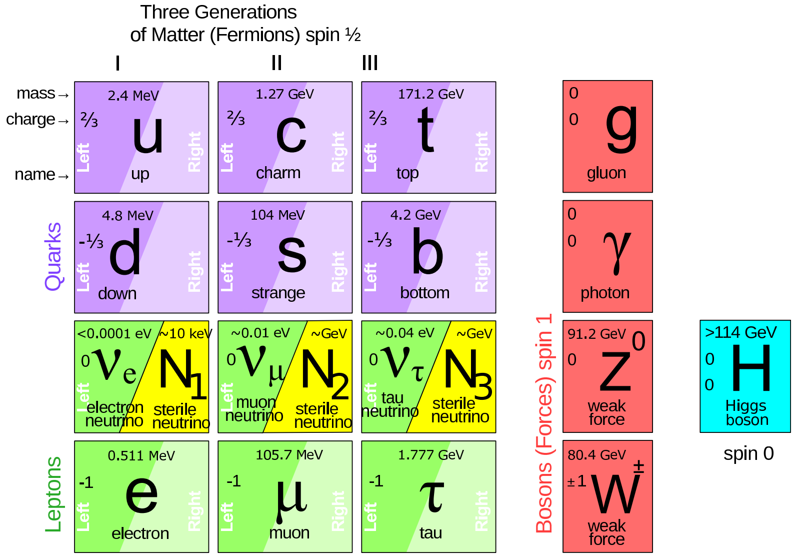
UV completion of the Standard Model

Minimal theory below M_P: νMSM





Role of N_1 with mass in keV region: dark matter

Role of N_2, N_3 with mass in 100 MeV – GeV region: “give” masses to neutrinos and produce baryon asymmetry of the Universe

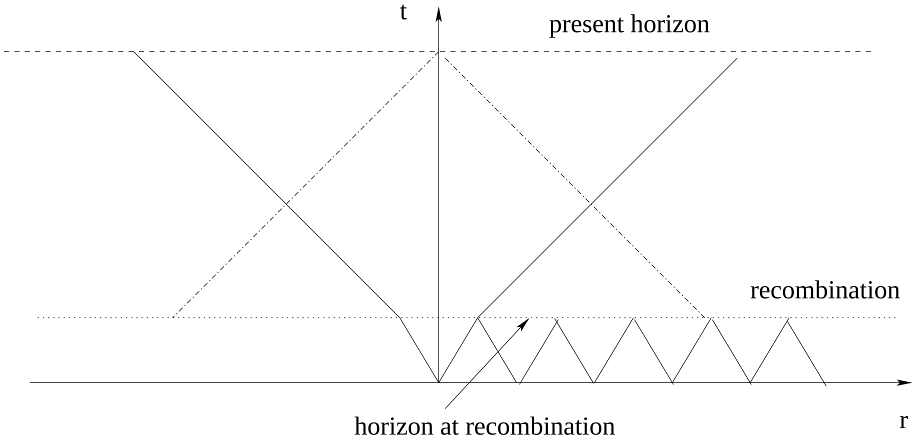
Role of the Higgs: give masses to quarks, leptons, Z and W and inflate the Universe.

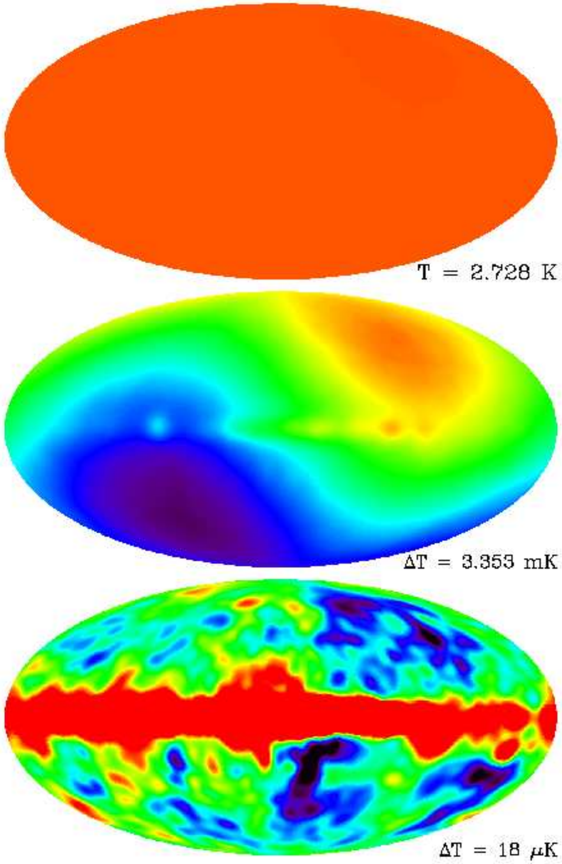
Back up slides

Inflation and the inflaton

Important cosmological problems:

Horizon problem: Why the universe is so uniform and isotropic?



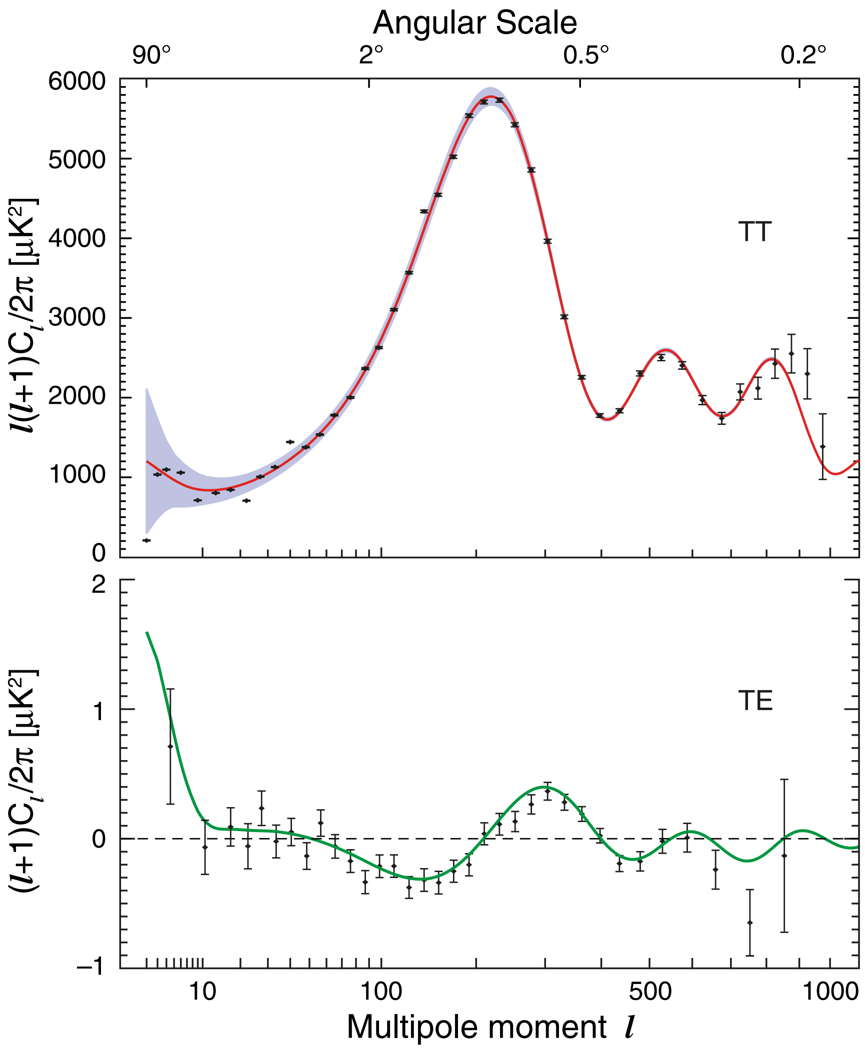


Expected fluctuations at $\theta \sim 1^o$:

$\delta T/T \sim 1$.

Observed fluctuations: $\delta T/T \sim 1O^{-5}$

Structure formation problem: What is the origin of cosmological perturbations and why their spectrum is almost scale-invariant?



Flatness problem: Why $\boldsymbol{\Omega_M + \Omega_\Lambda + \Omega_{\mathrm{rad}}}$ is so close to 1 now and was immensely close to 1 in the past?

All this requires enormous fine-tuning of initial conditions (at the Planck scale?) if the Universe was dominated by matter or radiation all the time!

Solution: Inflation = accelerated Universe expansion in the past

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Mechanism: scalar field dynamics

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Why scalar?

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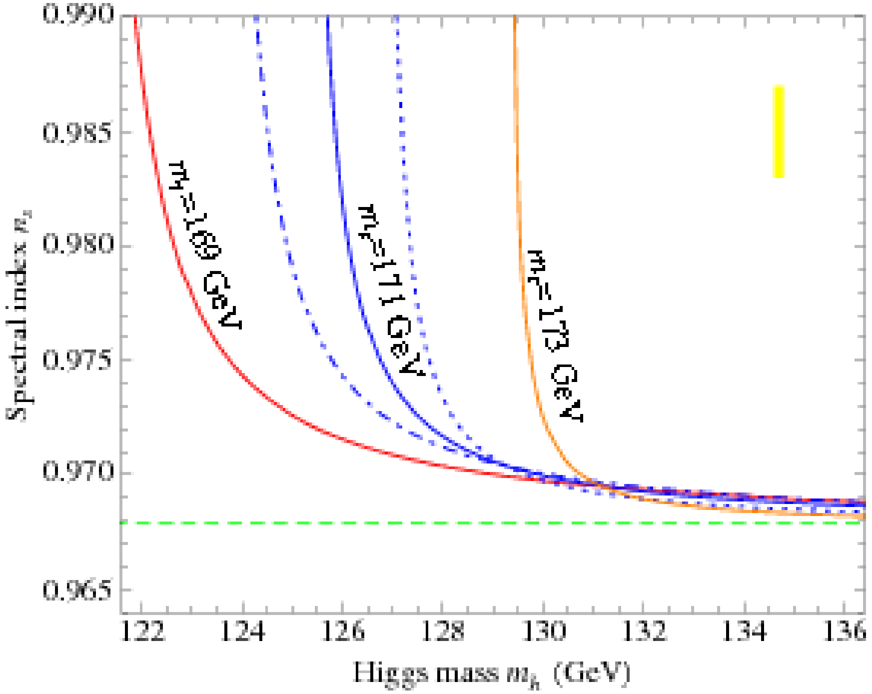
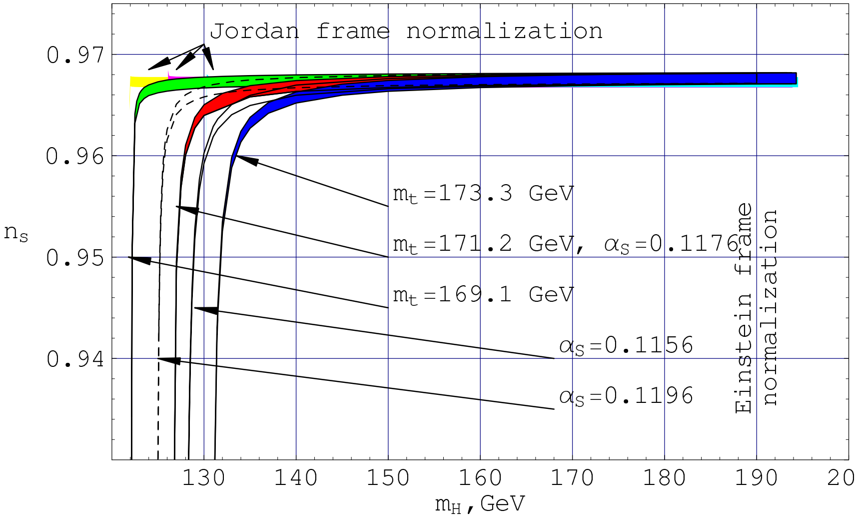
- Vector - breaking of Lorentz symmetry
- Fermion - bilinear combinations are equivalent to scalar fields
- Uniform scalar condensate has an equation of state of cosmological constant and leads to exponential universe expansion.

Comparison with other works

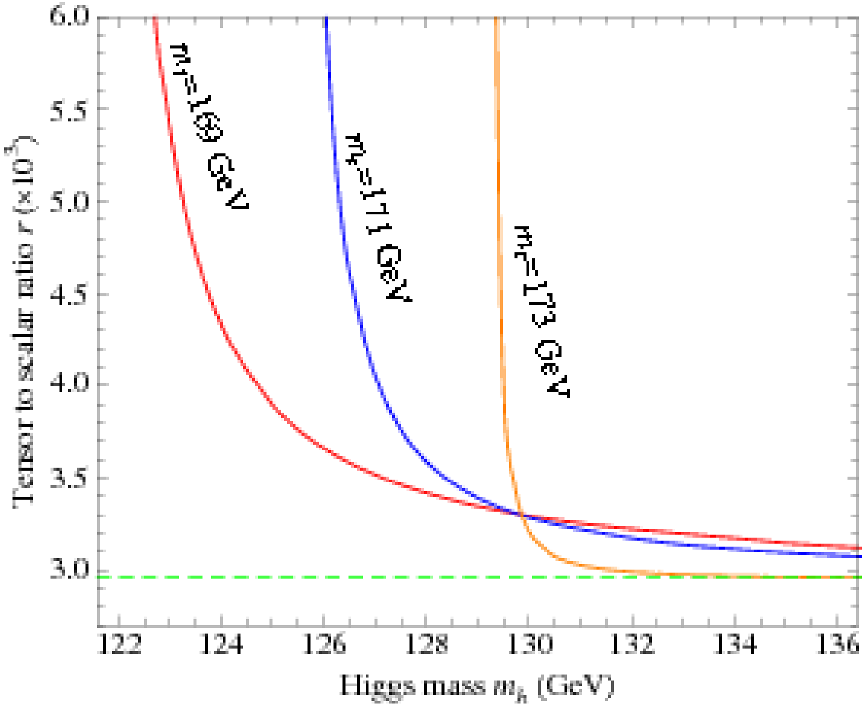


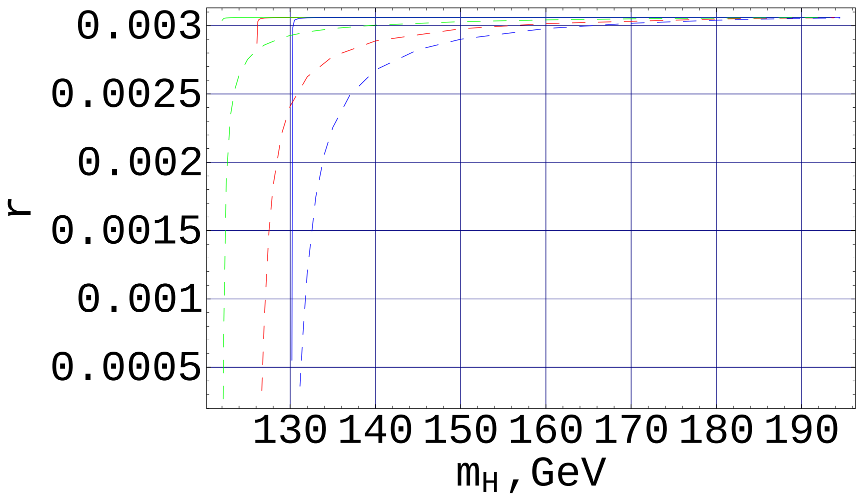


Simone, M. Hertzberg and F. Wilczek

Bezrukov, MS

Comparison with other works





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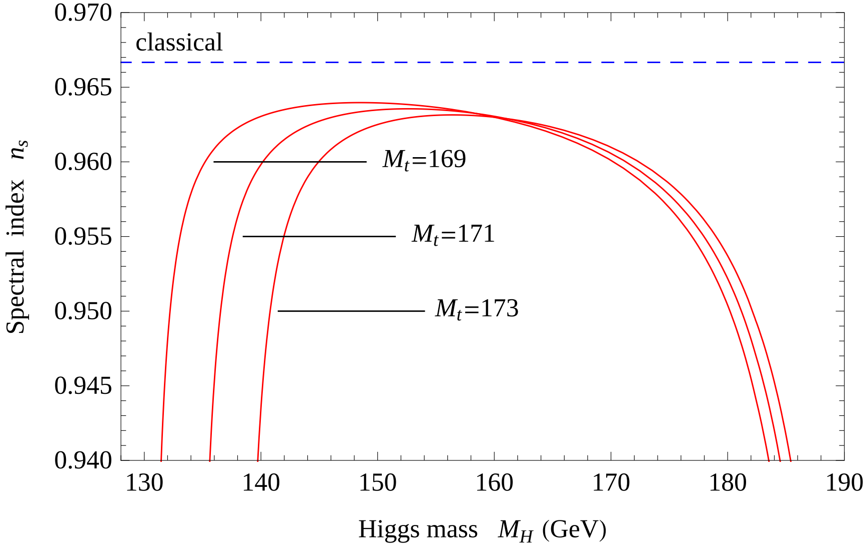
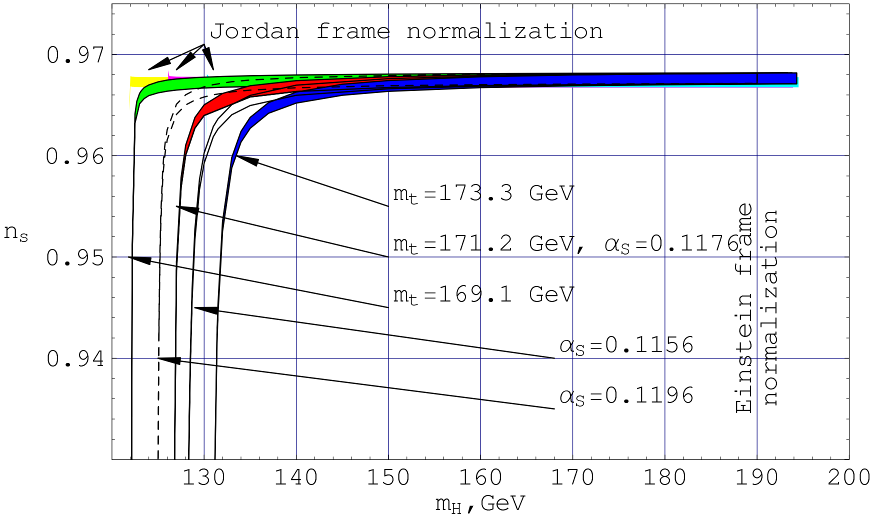


Barvinsky et al ’09



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Reasons for discrepancies

- Simone et al : Formalism is not gauge invariant. In particular, Landau gauge was used, but the contribution of Goldstones was not included. Consequences:
  - Running of coupling constants in the inflationary phase is different from ours.
  - Pole-$\overline{MS}$ matching conditions for coupling constants at the electroweak scale are gauge dependent.
- Barvinsky et al : The mass of Goldstone bosons in the inflationary phase was taken to be $m_G^2 = \lambda v^2$. In fact, it is

$$m_G^2 = \frac{\lambda v^2}{(1 + \frac{\xi v^2}{M_P^2})^3} \ll \lambda v^2$$

Consequence:

- the Goldstones contributions is largely overestimated (by a factor 50^6), what resulted in decrease of n_s for large Higgs masses and in larger deviations from the tree value at small Higgs masses.

Comparison with other works

Germani - Kehagias '10 : Yet another possibility for Higgs-inflation: action with derivative couplings,

$$\Delta S = \int d^4x\sqrt{-g}\left\{R^{\mu\nu}\frac{\partial_\mu h \partial_\nu h}{2M^2}\right\}, \quad R^{\mu\nu} - \text{Einstein tensor}$$

No new degrees of freedom are introduced as previously. COBE normalisation: $M \sim 10^{10}$ GeV. Different prediction of spectral indexes ($N \simeq 60$ is the number of e-foldings):

$$BS : n_s \approx 1/(2N) \approx 0.97, \quad r \approx 12/N^2 \approx 0.003$$

$$GK : n_s \approx 5/(3N) \approx 0.97, \quad r \approx 8/(3N) \approx 0.05$$

In GK case no analysis of reheating and of radiative corrections has been performed.